IV. (dis)Order of Operations: PEMDAS[1][2] and the Left-to-Right Rule

PEMDAS caveat: This **topic** is presented early on and is firmly interwoven into **school-math**. Its correction can prove difficult if not well explained. The best advice for **parents** is to read this section attentively, to at least **understand** the **labyrinth** the child is caught in. Depending on your situation you may want to follow the advice in the **What to do about it?** sections (**p...**). Pay special attention to the **left-to-right rule** as it will probably not be *fresh* in the memory of most **parents**, to say the least!

There are **flowcharts** in section **IV/2/ii/a** that will help to understand the full picture .

Section **IV/3** contains a **hack** to *avoid* **the whole thing**!!

This topic is so unfortunate that I find myself forced to divide it into *two* main sections: **IV/1. PEMDAS** and **IV/2. The left-to-right rule.**

Each of these is then divided into **What's going on?** (the *source* of the confusion) and **What to do about it?** (*practical* advice for dealing with the confusion and passing the **test**).

I recommend always reading the W**hat's going on?** sections first, or you simply won't understand what the issue is, *why* your child is distraught. But you can also skip straight to the **What to do about it?** section, if in a hurry.

1 In England and elsewhere the same thing is called **BEMDAS**. **GEMDAS**. **GERMDAS** etc.

2 **PEMDAS** almost seems to be meant as a warning affixed to the entrance to **school-math**: “Hey, don't get smart with us look at what we can unleash on you if you dare try. Be warned.” And many are.

IV/1/i. PEMDAS, What's going on?

PEMDAS is first presented in **5th** or **6th grade** and often creates confusions to last a lifetime. Alas, it would be no exaggeration to say that this is where math stops making sense for many students, in ***6th grade***!

For those of you who have been fortunate enough to completely scrub **PEMDAS** from your memory, it is *supposed* to represent the **order** in which we do **mathematical operations** such as **multiplication**, **division**, **addition**, **subtraction** and (for some reason) **exponents**.

Parentheses	**E**xponent	**M**ultiplication/**D**ivision	**A**ddition/**S**ubtraction
P	**E**	**M / D**	**A / S**

$$(3+4)^2-7\times 5=7^2-7\times 5=49-7\times 5=49-35=14$$

First the **parentheses**, then the **exponents**, then **multiplication/division**, then **addition/subtraction**

But what is really going on here? The basic kind of question **PEMDAS** is supposed to answer is:

$$5+3\times 7$$

Do I add or multiply first ?

I. add the 5 and the 3 *first* (8) and then multiply by the 7 to get 56

or

II. multiply the 3 and the 7 *first* (21) and then add the 5 to get 26

The answer is **II.** multiply the 3 and the 7 *first* and then add the 5 to get 26

To always multiply *first* when nothing like **parentheses** are involved is a **convention** (like a language) that has been agreed upon since we first started writing down **arithmetic**.

You always **multiply/divide** *before* you **add/subtract**. This is a fundamental and unalterable rule of mathematics!

Reversing the **order**

$$4+3\times7=25$$

$$3\times7+4=25$$

does *NOT* change anything here!

Note to yourself for later:

So this is not, I repeat *NOT*, when to use **the Left-to-right-rule!** because $\times$, $-$ are *not* **equal operations**, see **p..., IV/2/i**

If you wanted to write an **expression** for the other sentence, line **(I)**, then you would use **parentheses**!

This is the whole reason we have them:

I. *add* the 5 and the 3 *first* (8) and then multiply by the 7 to get 56 is written:

$$(5+3)\times7=56$$

Other examples of this are:

multiply first $3\times7-4=17$ subtract first $3\times(7-4)=9$

divide first $16+24\div4=22$ add first $(16+24)\div4=10$

So far so good!

I am currently homeschooling a **third grader** by **common core standards** and he just learned all of this so far. It is in his **3rd grade** textbook by HM[3]!

Parentheses are present and essential from the very beginning of **arithmetic** in **3rd grade**

So, lets just repeat what is clear and *natural*, what a **3/4th grader** already knows:

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3 Houghton Mifflin

<u>two basic rules:</u>

1. **Parentheses (brackets)** always go *first*.

2. Otherwise, **multiply** (or **divide**) *first* before you **add** (or **subtract**)

Sound good? In fact, this is all your child needs to understand....(**p...**)

At least until **exponents** magically appear and **school-math** makes up a weird corny rhyme ("**Please Excuse My Dear Aunt Sally**") that forms an **acronym** (**PEMDAS)** for a **process** that is at once unnecessary, confusing and, believe it or not, simply *wrong* in more than one way.

PEMDAS contains no new information, just the **two basic rules** above plus **exponents** and some very unnecessary confusions

To see how, read on.

Of course, school math creates **exercises** with **exponents** to fit its own teaching methods, so there is (almost) no way around this **wilderness of meaninglessness** and you will have to read the next section:

PEMDAS, What to do about it?

Or... you could always just skip directly to the **hack**!

IV/3. Plan B

Believe it or not, there are even mathematical **expressions** where **PEMDAS** simply does not **work**!

(p...)

The real question is...

Why do we need **PEMDAS** at all?

Take another look at the **two basic rules** from above.

1. **Parentheses** (**brackets**) always go *first*.

2. Otherwise, **multiply** (or **divide**) *first* before you **add** (or **subtract**)

3/4th graders already know these two things quite naturally, at least until they encounter **PEMDAS.** Ignoring the **E** for **exponents** for just one moment (see below), what *new* information does **P...MDAS** in fact contain? It contains a multitude of **confusions**, but it does not contain any *new* information!

All **P...MDAS** is saying is: **parentheses** *first* then **multiplication/division** *then* **addition/subtraction**.

That is exactly what was clear since **3rd grade** from the **two basic rules**!

Furthermore, **PEMDAS**'s claim to fame is that it represents the **order of operations**.

So, the fact that **M** for **multiplication** comes before **D** for **division** must mean something? Or could **PEMDAS** actually also be **PEDMAS**? We have the same confusion for **addition** and **subtraction**; could it just as well be **PEMDSA**? **PEDMSA**?

Of course *all* these **orders** are possible.

You certainly do *not* always do **multiplication** *before* **division**

and you certainly do *not* always do **addition** *before* **subtraction**

Equal operations:

Here **PEMDAS** does not tell us which **operation** to do first! (even though it sure looks like it does)

multiplication/division are called **equal operations**

addition/subtraction are called **equal operations**

Also **equal operations**:

multiplication/multiplication **division/division**

addition/addition **subtraction/subtraction**

Unequal operations:

These are all the *other* possibilities,

the ones where **PEMDAS** *does* tell us the **order**:

Multiplication/Addition **Division/Addition**,

Multiplication/Subtraction **Division/Subtraction**

are all the **unequal operations**.

Now you can call **PEMDAS** what you want: *untrue*, because it misrepresents the order between **equal operations**, or just simply *confusing*. But one thing is certain, if you ignore the **E** it is, in fact, unnecessary because the students have known the **two basic rules** above since **3rd grade**. (Or at least they thought they did, by this point many are understandably confused again).
Several gifted students have told me that they recall being confused about why **M** comes before **D** in **PEMDAS** for years. In the mind of a **6th grader** (or anybody really), if **PEMDAS** is *not* saying something about the order of **M**/**D** or **A**/**S** then what exactly *is* it saying? What is it for? As explained above, we've already known everything else since **3rd grade** (except the mysterious **exponents**). The fact that **PEMDAS** is meaningless, in the sense that it does not contain *new* information (except **exponents**), makes it even *harder* for students to believe/understand what exactly is going on here. It's like the perfect **confusion**, a masterpiece of self-perpetuating non-sense. *Genius*!

So the secret sauce must be in the **E** for **exponent**, right? That must be the reason kids are asked to memorize "**Please Excuse My Dear Aunt Sally**", (**PEMDAS**). So far, leaving out the **E**, we have learned absolutely nothing new from this **acronym**. What we have managed to do is to lose half the class because **M** does *not* always come before **D**, even though that's what **PEMDAS** absolutely seems to be saying (remember these are **5/6th graders**.)

6th graders indeed! They are learning how to do **basic arithmetic**, starting a long precarious voyage of the mind, first tender steps.

Why do **6th graders** need to start worrying about **exponents** while they are trying to understand **addition**, **subtraction**, **multiplication** and **division** for the first time?

What is that **E** doing there? How did it get there? Was it because some **acronym** enthusiast in the 13th century needed a vowel to make a "word"? At this point, that seems as likely an explanation as any for this truly crazy cacophony of concepts, and so we have **6th graders** being taught **exponents** while they are attempting to grasp **addition** and **multiplication**.

This monstrosity is universally hailed and repeated as the basis of understanding of all **arithmetic** and **algebra**. It is one of the first major mathematical **abstract principles** we introduce our poor children's naturally inquisitive and optimistic brains to.

An argument could theoretically be made that **exponents** are the **third level** of **operations** and therefore must be included in any attempt to codify **order of operations**. But first of all this is supposed to be a **teaching tool** for **6th graders** just learning **addition** and **multiplication** and not a mathematically complete system with a chance to win the *Nobel Prize*, and *second of all* if you are going to bring up **exponents** then

...what about (square) **roots** ??

Roots are the **opposite operation** to **exponents**.

That's the same thing as bringing up

addition without **subtraction**!

or **multiplication** without **division**!

PEMDAS is unquestionably a gross **violation** of mathematical thought.

A *new* rule, the **left-to-right rule**, **p...**, is about to explode onto the scene (like a **booby trap**, **p...**) to "clear up" the question of what to do first: **multiplication** or **division**. Not only is this rule (of course) not the order **PEMDAS** would seem to indicate (**M/D** and **A/S**) but, far more egregiously, this new rule *contradicts* the perhaps most important concept in **PEMDAS-** that idea that **parentheses** are supposed to be the way we determine the **order**.

In the **10th edit** of this sample, I have finally realized/remembered that:

$$2^{3^2}$$

defies PEMDAS. The **acronym** simply does *not* tell us *which* **exponent** to do *first* here!

And, *Yes*! It does make a difference:

$$2^{(3^2)}=2^9=512 \quad \text{or} \quad (2^3)^2=8^2=64$$

Using the **rule** suggested in this **chapter**, **p...**, the **exponent** applies *only* to the thing (**number** or **parentheses**) it is *directly above*. This is **unambiguous** and can *only* mean

$$2^{(3^2)}=2^9=512$$

which is the *correct* **interpretation** of this **expression**.

I do wonder how **school-math** treats this case. My guess is that it for the most part it is simply not mentioned or addressed. But I'm sure there are also instances of pretending**,** one way or the other, that it results from **PEMDAS**, possibly even coming to the incorrect conclusion (as using **the left-to-right rule** would entail!)

But perhaps more likely, is a sudden instance on **parentheses**, which obviously would beg the **question** why not use **parentheses** instead of t**he left-to-right rule**, **p...**in the first place!?

Please send any **examples** to whyeveryonehatesmath@gmail.com

4

4 I mention this to make clear how incredibly confusing this maze must be for a **6th grader**, if the author needs years and 10 edits to completely (*hopefully*!) come to terms with this *labyrinth*.

IV/1/ii. PEMDAS, what to do about it?

PEMDAS caveat: This **topic** is presented early on and is firmly interwoven into **school-math**. Its correction can prove difficult if not well explained. The best advice for **parents** is to read this section attentively, to at least **understand** the **labyrinth** the child is caught in. Depending on your situation you may want to follow the advice in the **what to do about it?** sections (**p...**). Pay special attention to the **left-to-right rule** as it will probably not be fresh in the memory of most **parents**, to say the least!

There are **flowcharts** in section **IV/2/ii/a** that will help to understand the full picture .

Section **IV/3** contains a **hack** to *avoid* **the whole thing**!!

Please read the previous section **PEMDAS, What's going on?** or else the following might not make a lot of sense. First, let's just repeat what is clear and *natural*, what a **3/ 4th grader** already knows! (**p...**):

<u>two basic rules:</u>

1. **Parentheses (brackets)** always go first.

2. Otherwise, **multiply** (or **divide**) *first*, before you **add?** (or **subtract**)

Sound logical? In fact, that is all you need! Or rather that would be all that you need, if **school-math** did not...

...make **6th graders** do problems with **exponents** at the very same time that they learn how to **add** and **multiply** in the correct **order**.

So what to do about the **exponents**? Well, sadly the child does have to memorize **"Please Excuse My Dear Aunt Sally"** and go through the whole **PEMDAS** thing.

6th graders need to **memorize PEMDAS** (and the rhyme) so they can at least **recite** both.

But I would not recommend using that as an **explanation** for your child.

I would simply *add* **one more line** (**3.**) to the **two basic rules** (above)

that have been clear since **3rd grade**:

1. **Parentheses** (**brackets**) always go first.
2. Otherwise, **multiply** (or **divide**) *first* before you **add** (or **subtract**)

now just add one new obvious line:

3. **Exponents** *always* apply *only* to the thing (**number** or **parentheses**) they are *directly above.*

Fixed!

$$(3+4)^2-7\times5=7^2-7\times5=49-7\times5=49-35=14$$

parentheses first, **exponent** applies *only* to the **parentheses** or **7** it is *directly above*, mult/div, add/subtract

What is the better solution?:

a) Try to justify a very confusing useless **acronym** that will forever haunt your child (memorize **"Please Excuse My Dear Aunt Sally"** without further explanation.) By doing this you will be placing yourself on the wrong side of a **5-12th grade** battle against **common sense**, trying to justify **school-math**.

b) Admit **PEMDAS** is nuts, but explain to your child that they will unfortunately have to memorize it and the silly **acronym** anyway. Then reassure them that all that's actually happening is that we are adding one obvious line about **exponents** to this natural list that was already clear. This way *long term damage* is hopefully avoided.

Teachers cannot tell how a child is *getting* the **order** so there will be no difficulties in that respect. When doing **PEMDAS** exercises students must simply do the **operations** in the correct **order**, often even just write down the correct answer. The students do not need to show *how* they get the **order** from **PEMDAS**.

Math is just so much easier when it makes sense to the child. But, please read the **PEMDAS caveat** at the **beginning** of the section!

I would obviously prefer to wait until the student has fully grasped **addition** and **multiplication** before throwing **exponents** at them. The reality is they will have to do the **exercises** presented to them in **school**. These **exercises** tend to have a lot of **exponents** in them because otherwise **PEMDAS** would not be necessary at all. (Thus proving the age old adage that the "primary function of a bureaucracy is to create more bureaucracy").

IV/2/i. The Left-to-right Rule, What's going on?

You'd like to think we are done with the confusions of **PEMDAS**? Watch the following collision of nonsensical ideas. This brazen “**anti-mathematical activity**”[5] is happening in real time every day to **6th/7th graders** worldwide!

Division/multiplication (and **addition/subtraction**) are called **equal operations**, **p...**

and remember **PE**(**MD**)(**AS**) doesn't tell us which to do *first*, **multiply** or **divide** (**add** or **subtract**). **p...**

So the question **the left-to-right rule** is now supposed to clear up is:

$$24 \div 8 \times 3$$

Do we do division or multiplication first here?

Depending on which way you do it, you often, **(p...**) get *2* different answers, for example:

$$(24 \div 8) \times 3 = 3 \times 3 = 9 \quad \text{and} \quad 24 \div (8 \times 3) = 24 \div 24 = 1$$

If you were to take **PEMDAS** at face value and always do **M** before **D** you would often get the wrong answer, **p...**

The following “rule” is without a doubt the beginning of many students', seemingly mysterious, fundamental insecurities with **parentheses**. Because, aren't we supposed to be using **parentheses** here? Isn't that what **PEMDAS** just taught us? Apparently not.

The **left-to-right rule** is the main reason parents are often humiliatingly forced to give up on helping their child in **math** around **6th grade**. Obviously, most adults have forgotten all about this since **7th grade** (the rule disappears in **8th grade**), so they are actually incapable of doing their child's **6th grade homework**!!

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5 Perhaps we will one day have our own **un-mathematical activities committee** (HUMC?)

The left-to-right rule ($6^{th}/7^{th}$ grade exclusively):

If we have two **equal operations** (**M/D**, **A/S**, **M/M**, **D/D**, **A/A**, **S/S**, **p...**) then we do the **operations** from **left-to-right**, like reading

So $24 \div 8 \times 3$ now *automatically* means $(24 \div 8) \times 3 = 3 \times 3 = 9$

not $24 \div (8 \times 3) = 24 \div 24 = 1$

and

$24 - 8 + 3$ now *automatically* means $(24 - 8) + 3 = 19$

not $24 - (8 + 3) = 13$

this **rule** applies *only* for **equal operations** and *not* for **unequal operations** like **M/A** or **D/S , p...**

(where **PEMDAS** *does* tell us the order)

See below two troubling and annoying tidbits about this rule. You don't really *need* to know this. But if you do happen to bump into it, you and your child might just throw up your hands in despair, wondering if you have perhaps misunderstood everything so far (as it happened to me, **p...**).

$$24 + 8 - 3 = (24 + 8) - 3 = 32 - 3 = 29$$

or $24 + 8 - 3 = 24 + (8 - 3) = 24 + 5 = 29$

For **addition** and **subtraction**, with **addition first**

THE **ORDER** MAKES *NO* DIFFERENCE!

Try it! **The left-to-right rule** *only* makes a difference for (**A**,**S**) when **subtraction** is **first**

$$24 - 8 + 3 = (24 - 8) + 3 = 16 + 3 = 19$$

$$24 - 8 + 3 = 24 - (8 + 3) = 24 - 11 = 13$$

$$3 \times 24 \div 8 = (3 \times 24) \div 8 = 72 \div 8 = 9$$

or $3 \times 24 \div 8 = 3 \times (24 \div 8) = 3 \times 3 = 9$

For **multiplication** and **division**, with **multiplication first**

THE **ORDER** MAKES *NO* DIFFERENCE!

Try it! **The left-to-right rule** *only* makes a difference for (**M**,**D**) when **division** is **first**

$$24 \div 3 \times 8 = (24 \div 3) \times 8 = 8 \times 8 = 64$$

$$24 \div 3 \times 8 = 24 \div (3 \times 8) = 24 \div 24 = 1$$

[6]

6 In the name of *mathematical sanity* it should be noted here that in the case of **addition** and **subtraction** the **order** actually never makes a difference if **subtraction** is re-written as: $24 - 8 + 3 = 24 + (-8) + 3$ first. But just try and explain that to a confused **6th grader** (I'm kidding *don't*). This way the **whole mess** does have an odd little symmetry that can be exploited see **IV/3.**, **Plan B**!

<u>Remember</u>

The left-to-right rule does *not* apply to **unequal operations** (like **A/M, p...**)

So $\mathbf{24+8\times3=24+(8\times3)}$ and not $\mathbf{(24+8)\times3}$

Do *not* use the **left-to-right rule** here

because when you have **unequal operations**, **multiplication** *always* beats **addition**.

This is where **PEMDAS** itself *does* apply!

I actually had to google this at one point myself because I got confused...wait does **the left-to-right rule** apply in this case or not?

Once **6th graders** have memorized **the left-to-right rule** it does not pose an *immediate* problem in itself for most kids, they simply execute the **series** of **commands** that is **PEMDAS** and the **left-to-right rule**. (see **flowchart p...**)

The fact that **the left-to-right rule** only applies to **equal operations** and not **unequal ones** does, understandably, initially lead to short term **confusion** and consternation, because even **6th graders** can tell that this goes against everything math is supposed to be at its core. This rule only applies sometimes, like opposite side parking Sunday between 5-7 pm. But, the whole mess is soon **memorized** by most students.

For many students executing the **Order of Operations**, once memorized, can make them feel safe in math, a very natural reaction. But it is a false sense of security that lulls them into believing this is how math works. This is *not* how math works. Math is more than blindly repeating instructions that don't make sense if you think about them too much. In fact, it is the exact opposite! Not to mention that **the left-to-right rule** will actually stop existing very soon (in **8th grade**), unfortunately right along with that false sense of security.

The real problem is *long term* as this is where **parentheses** should be firmly grasped, instead we are literally doing the **opposite** of what we just "taught" them about **parentheses** in **PEMDAS**.

This is where many very young kids simply give up respecting the natural **beauty of math** and come to the conclusion that **mathematics** is nothing more than a meaningless exercise in **memorization**, full of **contradictions**, and it is better to not try too hard to make sense of it.

There is, of course, also the perfectly logical and correct way of doing this using **parentheses** (and this is how **mathematics** actually works forever-after **7th grade**):

We *should* just use **parentheses** and write either:

$$(24\div 8)\times 3=3\times 3=9 \quad \text{or} \quad 24\div(8\times 3)=24\div 24=1$$

depending on whether we want to **divide** *first* or **multiply** *first*.

This would certainly be in line with **PEMDAS** as the first letter **P** stands for **parentheses** meaning they always go *first*, no matter what. **Parentheses** or **brackets** are, of course, one of the most important concepts in math altogether. And even though one would think they would be one of the easiest tools to master in math, for some mysterious reason many decent students are consistently insecure about **parentheses** all the way through **high school**.

Using **parentheses** here would be great, it would be the best! Unfortunately, that is not what happens in **school-math**...quite to the contrary. There is even a **school-math barricade** to keep you from fixing it yourself, **p...**

To sum it up:

After making up a confusing, if not incorrect, **acronym** (**PEMDAS)** for something[7] **3rd**, **4th**, and **5th graders** thought they understood (and probably did until **PEMDAS** happened and **exponents** suddenly magically appeared), **6th grade school-math** with the **left-to-right rule** now refuses to follow the one piece of actual solid advice contained in **PEMDAS**, **parentheses**.

The left-to-right rule is necessary because **PEMDAS** does *not* tell us whether to **multiply** or **divide** *first* , even though it sure looks like it does! **p...** . This should be resolved by using **parentheses** as is always later the case in **mathematics**. There is no good reason not to. And yet, for some unknown reason, and despite the prominence afforded the concept of **parentheses** in **P-EMDAS**, **6/7th grade school-math** does not use **parentheses** to clear this up. Instead **the left-to-right rule** creates a clumsy, anti-mathematical mess and confusion that often lasts a life-time.

Finally, in **7/8th grade** as a grand finale, **school-math** abruptly and silently abandons the **left-to-right rule** with absolutely no further **explanation**, as if it never happened. It becomes a fleeting memory, remaining stuck somewhere in the students' math sub-conscience like a scalpel a doctor forgot in a patient after an operation. Search deeply enough and you too will probably vaguely remember something about reading math left-to-right? What happened to that?"

What happened was that **the left-to-right rule** was not math in the first place, and so it had to be *abandoned* in **8th grade** because it contradicts actual mathematical notation.

Even mathematicians I have spoken with don't recall this absurd made-up **school-math rule**, and are puzzled by **6th grade homework**.

This really is a great way to start things off and get on the right foot with the students and their natural interest in math.

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7 **Two basic rules p...**

Also tricky:

In both the **A/D** and the **M/D** cases only *some* of the cases are affected by **the left-to right rule** at all! , **p...**

I would probably not mention this last fact to your child unless it comes up by itself. Or if you choose to go down the route outlined in **IV/3**. **Plan B***,* **PEDMSA**.

Once when I was tutoring a **sixth grader** for the first time, with the mother observing, I attempted to explain the **the left-to-right rule** to the mother. I happened to chose an **M/D** example where **multiplication** came **first**. I could not, for the life of me, figure out what was going on. Why was I getting the **same answer**, as if the **order** *didn't* matter? Had I made the whole thing up? I broke out in a cold sweat for fear of embarrassing myself and maybe even losing the client! It wasn't until hours later, when I finally had 15 minutes to sit down and chew the whole thing through that I figured out what was going on!

If this is what happens to a 45-year-old who majored in math at *UC Berkeley*, imagine what it does to the mind of a **sixth grader** left all alone with this stuff and with no one around to point out the ridiculous maze she/he is in.

The **Common Core** nowhere mentions **PEMDAS** but does say:
CCSS.Math.Content.6.EE.A.2.c
"... perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations)."
So at a minimum they seem to be keeping **the left-to-right rule** and **exponents** in **6th grade** alive, although this is pretty darn ambiguous (are they talking about **equal operations**[8] or all operations?)

Now what can you do about it?

8 Oh, **Common Core**! I really had no problem with you to start off with. This is obviously referring to **the left-to-right rule**, but they can't seem to bring themselves to say it, even with a gun to their head. It almost sounds like they are saying we should not be using **parentheses** yet, which is ridiculous since **parentheses** have been around since **3rd grade**. This is obviously a concession to the powers that be, the notorious **left-to-right rule** cartel strikes again, Status Quo!

IV/2/ii. The Left-to-right Rule, What to do about it.

Please read the **previous section** or you may not understand the following

Had enough? You could make the **change** to a much simpler but mathematically ridiculous **hack** that does not even pretend to have a meaning. It takes care of everything including **the left-to-right rule** (which never comes up again in **school-math**) in one fell swoop.

IV/3. PED-M-SA, Plan B (p...)

If you continue to follow the path of **school-math** this is the situation:

The left-to-right rule cannot be avoided at this point in **school-math**

The best course is to explain to the child that the **left-to-right rule**

is not *real* **mathematics** and that it will *go* away after **7th grade (p...)**

At least you now **understand** what **confusion** the child is dealing with.

Remember, the **question** is: what does the following mean?

$$24 \div 8 \times 3$$

We *should* of course (as is always true from **8th grade** on)

just use **parentheses** and write *either*:

$$(24 \div 8) \times 3 = 3 \times 3 = 9$$

$$or \quad 24 \div (8 \times 3) = 24 \div 24 = 1$$

to indicate clearly whether we **divide** or **multiply first**

BUT IN **6/7th GRADE** WE DON'T

instead the **left-to-right rule**, **p...** says what to do *first*

The next best solution would seem to be, have the **6th graders** put **parentheses** in the correct place following the **left-to-right rule** themselves, and then work out the **problem**. This would minimize the **confusion** and *long term* damage to their understanding of **mathematics**. So:

When they see $24 \div 8 \times 3$ have them **correct** it to $(24 \div 8) \times 3$

and

When they see $24 \times 8 \div 3$ have them **correct** it to $(24 \times 8) \div 3$

After all, the first letter **P** in **PEMDAS** stands for **parentheses** because they always go first, no matter what. This is one of the most important **concepts** in **math** altogether.

Using **parentheses** here would be great!

Alas, this is no working **solution** either because **6/7th grade school-math**, in its particular brand of ingenuity, is filled with **exercises** that make this impossible. These **exercises** that defy this common sense solution appear only at this grade level to practice/justify this **anti-mathematical** rule.

For example:

$$3 \times 4 \div 5 \times 8 \div 9 \times 3$$

This expression leads to incredible confusion if **parentheses** are used to show what the **left-to-right rule** now tells us to do:

$$(((((3 \times 4) \div 5) \times 8) \div 9) \times 3)$$

I challenge the reader to find such an **expression** in any textbook *after* **7th grade** when **parentheses** are used correctly again. There may be a rare case here and there to test the students' understanding of **parentheses**, but that is precisely the point. These are involved **rare cases** that serve only to underline the necessity for intuitive **notation**-the *last* thing any mathematician wants to do is make up new and different **notations** for **rare cases**!

What would be a real world **word problem** for the expression above[9]?

If I make **3 ice cream cones** per hour for **4 hours** and then divide up all those ice cream cones amongst **5 friends** who then each get **$8 per cone**, then each split that money with their **9 siblings**, who then each buy **3 candies for $1**..how many pieces of candy would *each* of the **9 siblings** of the **5 friends** then have?

Seem reasonable? An everyday **word problem** for any **6th grader**. And even if they did get such a crazy problem they would never write it down in correct (or **left-to-right rule**) **notation**; they would do it step-by-step, which would be easier anyway.

The point is, this **type** of **expression** is an absurd **rare case** that really shouldn't be here. Correct me if I'm wrong, but it does seem pretty obvious that the only reason for these particular exercises is in fact to justify the existence of **the left-to-right rule**.

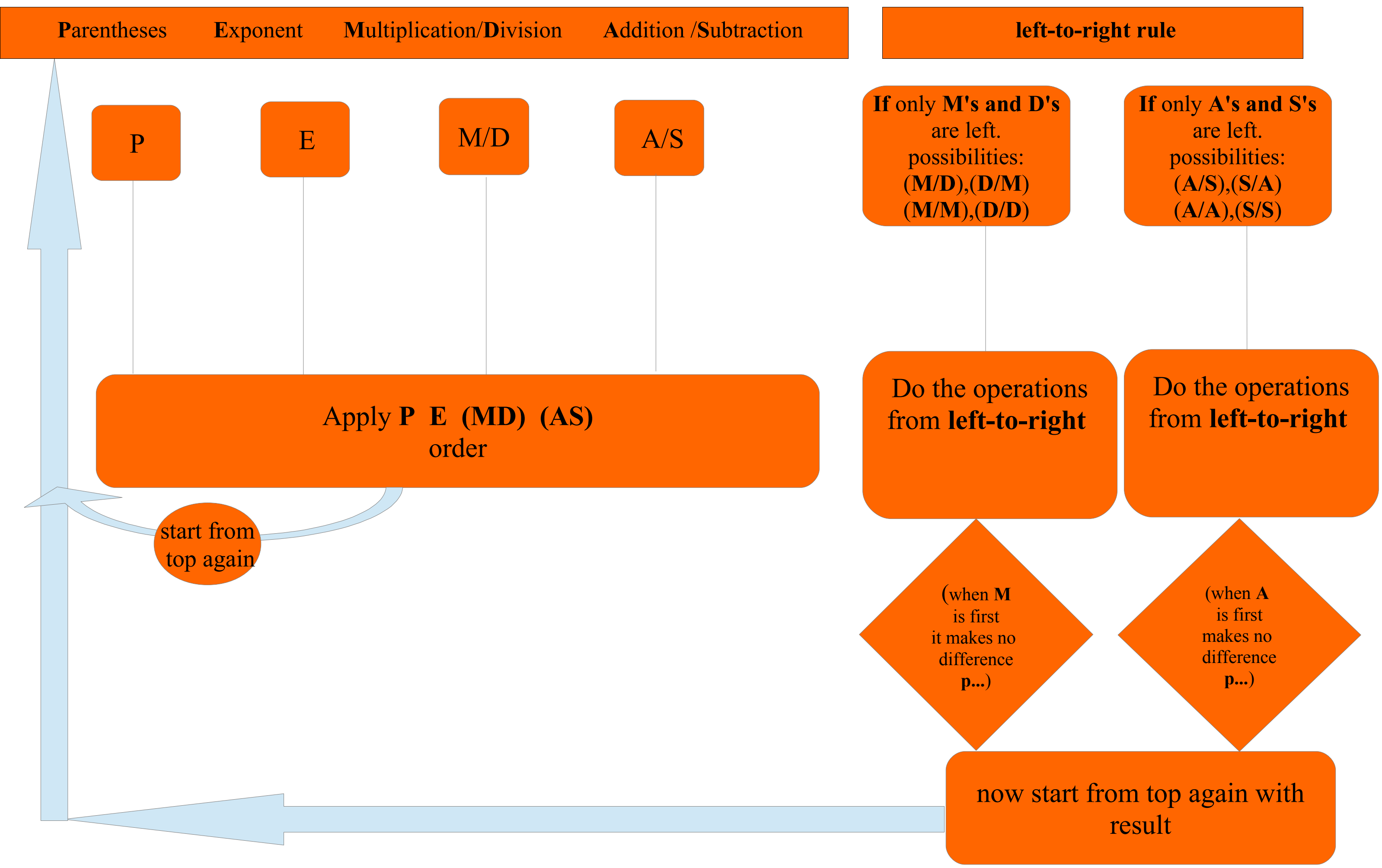
9 You know, the kind of "math is all around you", make math more relatable TED talking point.

IV/2/ii/a. PEMDAS Flowchart

Sorry, there is no way around this (well, actually...see **IV/3!!**) Still confused? Here is a step-by-step **flow chart** for getting through the maze that is **PEMDAS** and **the left-to-right rule**:

PEMDAS: FLOW CHART

unequal operations, **p...** | equal operations (**M/D** or **A/S**), **p...**

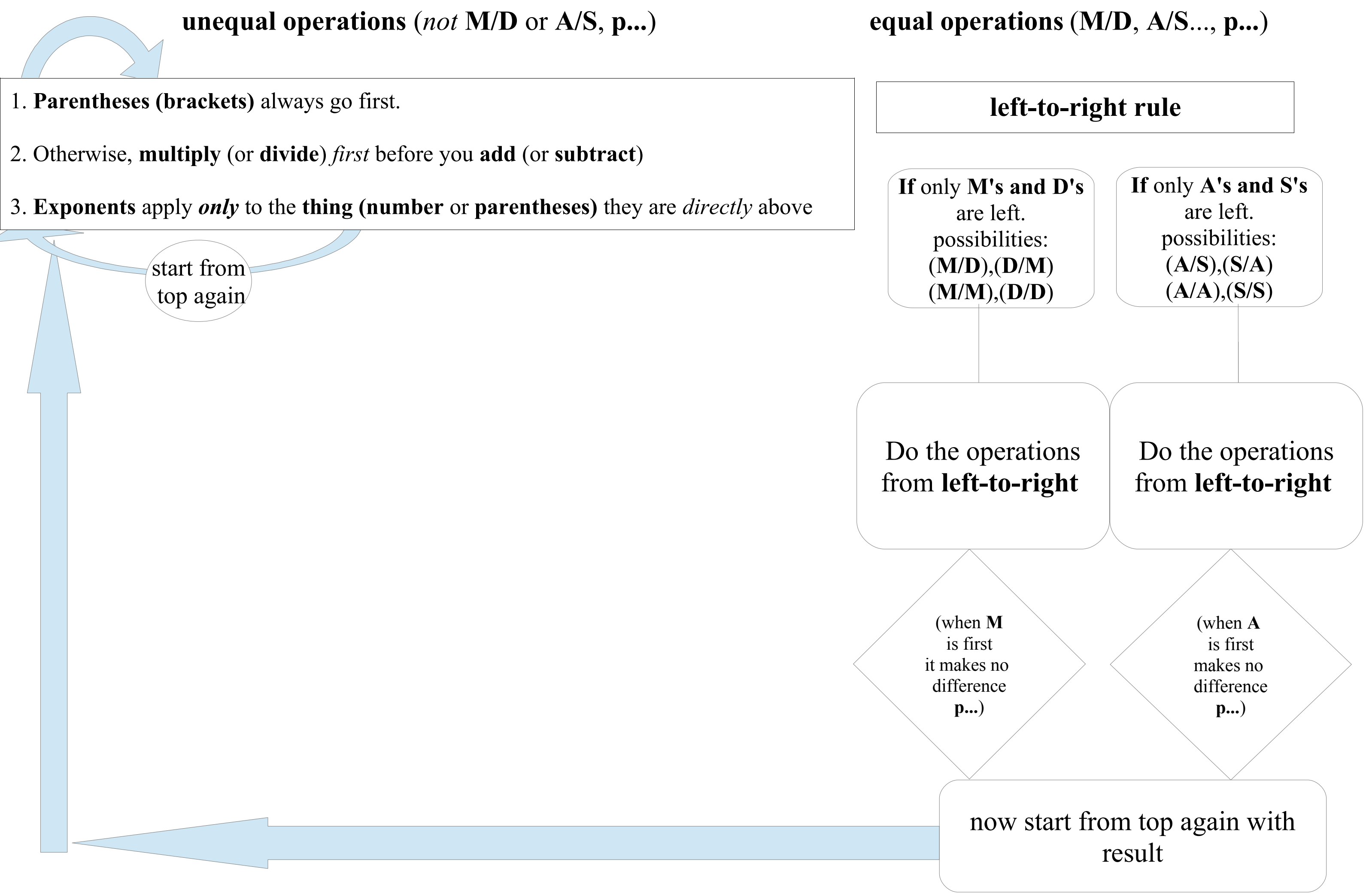


$(3+4)^2-16\div8\times2=7^2-16\div8\times2=49-16\div8\times2=49-2\times2=49-4=49-4=45$ First parentheses, then exponents, then **division or multiplication?** Do division *first* then multiplication because of **left-to-right rule**, finally do subtraction

There are millions of **PEMDAS exercises** with **answers** online (google: PEMDAS exercises with answers PDF), so I won't list more here (for now).

If you follow the advice in this chapter and don't bother (too much) with the **acronym** (**PEMDAS**) you unluckily still have basically the same crazy **flow chart** because of the presently (until **8th grade**) unavoidable (well, actually...see **IV/3. Plan B**!) **left-to-right rule**. But, because the only new thing that happened since **3rd grade** (asides from **the left-to right-rule**) is now one new easy line (**p...**) (**exponents**), the student hopefully understands the **order of operations** (asides from the **left to right rule**) by **common sense** and not by **rote memorization** of a diddle.

ORDER OF OPERATIONS: FLOW CHART



$$(3+4)^2-16\div8\times2=7^2-16\div8\times2=49-16\div8\times2=49-2\times2=49-4=49-4=45$$

First the **parentheses**, then **exponents** apply *directly* to the **7,** then **division vs multiplication** (do division first and then multiplication because of **left to right rule**) , then finally **subtraction**

For the record,

After **7th grade** we start following the fundamental rule that is true *forever-after* in **mathematics**:

Always use **parentheses**! No **ambiguous cases**! In other words:

$$(24 \div 8) \times 3 \quad \text{or} \quad 24 \div (8 \times 3)$$

but never the **ambiguous case** $24 \div 8 \times 3$

(**ambiguous case** means it is unclear what to do first, a **syntax error**, an **invalid command** such as: $3 \times \times ++4$)

so no more **left-to-right rule** ever again.

This is what the "**flow chart"** looks like after **7th grade**:

1. **parentheses** always go first.
2. otherwise **(M/D)** before **(S/A).**
3. **exponents (and roots)** always apply to the thing

   (number, parentheses, other exponents or roots, integrals etc.)

   they are *directly above*.

There is no need for a **flow chart** because it's **mathematics**, not the **DMV**!