

On separate scratch paper:

calculate  $b^2 - 4ac$ . If this is a very big number  
you may need to use a **calculator** next.

If  $b^2 - 4ac$  is a  
**perfect square**  
like  $\sqrt{49} = 7$

If  $b^2 - 4ac$  is **negative** or **not a perfect square**  
such as **-3** because it's negative  
or **46** (because  $\sqrt{46}$  doesn't "work") the **quadratic is**  
**prime**. Write down  $a \cdot c$  scribble as if you were *trying out*  
**factors** of  $a \cdot c$ . Write "the quadratic is prime."

If  $b^2 - 4ac = 0$   
just continue with the **QF**  
or  
save time with the **binomial theorem**

complete the **rest** of the **Quadratic Formula**.

Just take the **square root** of what you have already worked out  $\sqrt{b^2 - 4ac}$   
and since that always just gives you **D**, an **integer**,  
(or you should *not* have reached this **step** here). The rest is *easy*!

$$\frac{-b \pm D}{2a}$$

Do it **twice**. Once with the + and once with the -

These **two answers**  $r_1$ ,  $r_2$  are called the **roots** of the **quadratic**.

recognize the **Binomial Theorem**  
the **quadratic** will be of the form  
 $a^2 \pm 2ab + b^2 = (a \pm b)^2$   
such as  $x^2 - 6x + 9 = (x - 3)^2$

write down  $a \cdot (x - r_1) \cdot (x - r_1)$  and **multiply** by the **a** to get *rid* of all **fractions**.

Let's say **a=6** and  $r_1 = \frac{1}{2}$  and  $r_2 = \frac{1}{3}$  :

$$6(x - \frac{1}{2}) \cdot (x - \frac{1}{3}) = 2 \cdot (x - \frac{1}{2}) \cdot 3 \cdot (x - \frac{1}{3}) = (2x - 1) \cdot (3x - 1) \text{ the quadratic is factored!}$$

The *amazing* thing is this  
will *always* work! To see  
the reason/proof *why*!!  
**p...** It's **Group Theory**!

**foil** the **factored form**, but *don't* add the x's together. Now you have the **correct grouping (trick IV)**.

$$(2x - 1) \cdot (3x - 1) = 6x^2 - 3x - 2x + 1$$

on the answer paper

**multiply a and c**,  $a \cdot c$ , and **write it down** at the top of your **answer**. Now, leave room for **scribbling**, as if you were  
*trying out factors* of  $a \cdot c$ . Now, **write down** the **correct grouping** from above. Below that, **write the factored**  
**form** of the **quadratic**. Done!

On separate scratch paper

1. **calculate**  $b^2 - 4ac$  . If this is a very *big number* you may need to use a **calculator** next.

If this **number** is **negative** or *not* a **perfect square** then the **quadratic** won't **factor** into **integers** and is called "**prime**". But, if this **number** *is* a (**non-negative**) **perfect square** you *will* be able to **integer factor**, and the **quadratic** is *definitely not prime*!

(this is *not* obvious at *all!*, p...)

So now you already know *whether* this is a so called "**prime**" **quadratic** or *not*!

If  $b^2 - 4ac = 0$  use  $(a^2 \pm 2ab + b^2) = (a \pm b)^2$  (**Binomial Theorem**) to **factor** and save **time** or just *continue* here and the **two roots** will be just be **equal**, see below.

2. **complete** the **rest** of the **QF** to get the **roots**:  $r_1$  and  $r_2$  . (each **root** should be either an **integer** or a simple **fraction**  $\frac{a}{b}$  )

3. **write down**  $a \cdot (x - r_1) \cdot (x - r_2)$  .

If  $r_1$  and  $r_2$  are **integers** you are *done*. Of course this means **a=1** and you should just have used **trick I**, but no harm done.

Otherwise, **break up a** into the (**number**) **factors** necessary to make everything **integer**. You now have the **factored form of the quadratic**. This will *always* work if you passed **step 1**.

4. **Foil** the **factored form**, but don't add the **x's** together. Now you have the **correct grouping** for **trick IV**.

On your answer paper

5. **multiply a** and **c** and **write down** the result at the top of your **answer**. Now, leave room for scribbling as if you were **trying out factors** of  $a \cdot c$  . **Write down** the **correct grouping** from **step 5**. Below that, **write the factored form of the quadratic**. Done!