# Competitive analysis

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# The research on the myriads of school-math textbooks was done in bookstores and so it was cut short due to the pandemic. This is a very beta version, but it does contain many useful reviews if you are interested. It's just not very well organized or polished yet-to say the least!

First of all, I would like to name the school resources I *do* recommend to students 4-12. There are really only two. The *Fabulous Guide* for the ACT/SAT by the Applerouth company and (with some reservations, but nonetheless) Danica McKellar's whole series. The *Fabulous Guide* is just for the ACT/SAT so I do not even review it here, but it is an example of a very good school-help book.<sup>1</sup>

You can skip straight to the analysis of amazon rankings below, p...

At first, to get a gist of what the accepted thinking on this question is, I googled "why is math so hard?" which brought up a wealth of articles. These results list various supposed reasons why math is hard such as: anxiety, not enough time, preconceptions about math, dyslexia, numerosity (? WP) and so on. *None* of the reasons listed (even the Washington Post piece, "Why is math so hard for many"<sup>2</sup>) even mention the mathematical *content* of the *material* itself and its *presentation* in school, in the following to be referred to as *school-math*.

I then added the word "book" to my search which lead me to the following academic publication. Again, the theme of my book, the *math trauma*<sup>3</sup> caused by the *disastrous* school-math material, is not mentioned in this work it seems.

## Why math is hard for some children #431 in mathematics study and teaching (amazon)

Here is its abstract:

*Why Is Math So Hard for Some Children?* is the first definitive research volume that explores the evidence base for students' difficulties with mathematics. This landmark resource gives educational decision makers and researchers in-depth theoretical and practical insight into mathematical learning difficulties and disabilities, combining diverse perspectives from fields such as special education, educational psychology, developmental psychology, cognitive neuroscience, and behavioral genetics. More than 35 internationally known contributors share their expertise on

- 1. indicators of mathematical difficulties and disabilities
- 2. risk factors for poor mathematics outcomes
- 3. connections between mathematics and reading disabilities
- 4. neuropsychological factors in mathematical learning disabilities
- 5. information processing deficits
- 6. individual difference factors in mathematics difficulties
- 7. math anxiety

<sup>&</sup>lt;sup>1</sup> That apparently though is going out of print. This is odd but the tutoring company Applerouth behind the book is not very good, it is just these two books that are a cut above the rest.

<sup>&</sup>lt;sup>2</sup> https://www.washingtonpost.com/news/answer-sheet/wp/2013/03/27/why-is-math-so-hard-for-so-many/

<sup>&</sup>lt;sup>3</sup> Math trauma, Definition: The inability to successfully focus on a mathematical concept as it leads directly to emotional pain associated with previous efforts to open up and attempt to understand school-math that resulted in abject failure, prolonged confusion and finally despair. A compounded, cumulative disorder which slowly manifests itself over the course of many years of school-math

#### 8. the role of genetics

#### 9. effective instructional interventions

Comprehensive and multidisciplinary, this book gives readers the foundation they need to advance research, teaching strategies, and policies that identify struggling students and help put them on the path to stronger math skills.

## Amazon:

Here are some of the major categories of math books on Amazon:

## math bestsellers:

Most math bestsellers fall into the following categories, basically in this order: general/philosophy, economics, math in history, AP/higher level school-math books (calculus sells well), standardized exam prep, military schools, workbooks with only exercises, math story books, cheat sheets, using math in everyday life books. It takes almost until #70 to encounter the first math-help book!

Of the first 100 math bestsellers here are the math-help books that are not just exercise workbooks:

#69 for dummies statistics
#73 For dummies calculus (p...)
#74 statistics school-book
#77 statistics a graphic guide
#94 no-nonsense algebra (p...)
#95 self-teaching algebra (p...)
#97 painless geometry (p...)

Even though the <u>Big Fat Math Notebook</u> (p...) has an amazon rank of #2 in children algebra books, #5 in children general study aids, #1 in children fractions books and Danica McKellar's *Hot X* is ranked at #7 in the teen and young algebra category (asides from being highly touted by the NYT) they do not make it here. This fact really tells you how much more general-interest math books sell than math-help books which seems odd.

I treat all these books separately at the indicated page numbers, except the statistics books. Statistics is just a collection of methods with very little conceptual math at all. In fact, statistics is the one so-called math topic that really is more memorization than anything else, there is almost no explanation at all to these methods (in school). This probably explains their preponderance in this bestseller list, because this is the one topic where a mere collection of more or less decently organized cliff-notes is in fact all you need to succeed.

So in order to see what is really going on in the, apparently not very popular (go figure), mathhelp category, we have to be more specific:

General study aids:

children's math:

## Children's algebra:

## featured math categories on Amazon:

As a quick perusal of these list reveals, the vast majority of all math *help* materials end in 8<sup>th</sup> grade. This is a phenomenon I observed in Germany as well (see Bio, p...) There was a time in the 1970s when there was a lot of talk in the education community worldwide about making math more friendly, but it was all about grade school (Sesame Street comes to mind). This could be the remnants of that. Also, since the education community seems to have mutually agreed to not see the school-math *mathematical* material itself as the problem, almost all mathhelp books are written for lower and middle school children who purportedly need to be piece-fed and coaxed into math. This is really quite remarkable in itself. By this thinking, school-math is perfect. It is only the young children who need convincing, who need to be lured in. But once they reach 9<sup>th</sup> grade, they should really just get it.

Of course the fact that the only 3 real math-help books that make it into the top 100 bestsellers on Amazon (see above) are in fact for algebra and beyond (9-12<sup>th</sup>), seems to tell a completely different story!

The notable exceptions to this dearth of 9-12<sup>th</sup> grade math-help material are explored below:

For Dummies (p...)

The Idiots Guide (p...)

Demystify (p...)

Danica McKellar (p...)

## amazon best sellers trigonometry

A good way to see what I mean is to go to the best sellers in trigonometry.

Asides from a cardboard fold out with trig facts, like a cheat sheet, only two of these books are anything but school text books that just repeat school-math with all its flaws. If there was a great book about high-school math with a better approach to trig, shouldn't it show up here at the top?

Again, almost the only math-help for grades 9-12 seems to be the *For Dummies* series and indeed it shows up at rank #32 here:

Trigonometry-Workbook For Dummies

Why this book doesn't rank higher, when it is almost the sole irreverent-looking option here, I discuss below in the *For Dummies* section (p...). There is also *Trigonometry* by *Straight Forward Math* that is not a school textbook, but it is only ranked #132 here and seems to be more of an assembly of facts (cliff-notes) than anything actually elucidating.

## Best-Sellers-Books-Algebra

The same sparsity is apparent in the category algebra, although there *seem* to be a few options here.

## #6 painless algebra

Again, *For Dummies* is almost the only non-textbook, non-standardized exam book. This time ranked #96:

## Algebra-Workbook For Dummies

The Idiots Guide, Algebra comes in at an unbelievable #1379 in algebra and trigonometry.

The other irreverent-looking book in 9<sup>th</sup> grade algebra is the very different Danica McKellar's *Hot* X at #7 in the teen and young algebra category that I treat below in a separate section (p...).

Then there really not much in the way of math-help for 9-12. This in itself is a pretty shocking discovery. I hardly believe it as I write. I'll check again, and again...and again. Why?

I googled math-help 9-12<sup>th</sup> grade/high school and compared that to math-help 4-8<sup>th</sup> grade and it is simply a fact! Google cannot come up with a list of math help-books for 9-12<sup>th</sup> grade/high school, they just come up with websites and videos (Khan Academy), while math-help 4-8<sup>th</sup> grade immediately generates a colorful seemingly endless bounty of books!

I will now proceed with the books *explicitly* for grades 5-8. For Dummies, McKellar, the Idiots Guide and Demystified all have books that are for these grades but then go slightly beyond usually to 9/10<sup>th</sup> grade. I treat each of these series separately in their own sections in pages .....

There are a number of very grave problems/mistakes in school-math that I treat in the WEHM manual. I call them pitfalls, mathematical booby traps, webs, labyrinths, quicksand and so forth (WEHM, p...) These are things that come up in school-math on a regular basis, that basically blow up, envelope, suck in or gum up the student as they attempt to actually understand rather than submit to blind memorization. The period of resistance on the part of heroic students unfortunately mostly does not last very long without considerable outside positive influence.

I will use the most common and worst of these traps as *litmus tests* as I review the books. The easiest and fastest way to understand what is meant by this is *parallel lines and the transversal* (WEM, p...)



## Big Fat Math Notebook

amazon rank: #2 children algebra books, #5 children general study aids #1 children fractions books

A student of mine heard I was looking for study guides comparable to mine and came back from her room with a very promising looking book called "Big Fat Notebook" or "Everything you need to ace Math in one Big Fat Notebook". It turns out this is a best seller on Amazon in the math help category. At first this looked very much like what I am suggesting, and indeed I do love the format of a student's orderly notes written on lined paper or as their moto goes "notes from the smartest kid in the class". Just like my book it is clearly divided up by topic and the topics are presented in a non-wordy fashion, actually there is almost no explanation at all. It is clearly a reference manual to help with school-math. But that is where the similarities to the WEHM manual end.

The actual content of the topics is just a perfect carbon copy of what is presented in school with no clarification. In fact, a quick inspection found blind repetition some of the worst school-math offenses such as the vertical line test (WEHM, p...) and even a terrible mathematical *mistake* while repeating everyone's favorite acronym, PEMDAS. On page 197 this book states that exponents are to be performed *left-to-right*. This is one of the problems of PEMDAS that I point out in the WEHM manual (WEHM, p...). PEMDAS simply does not tell us how to do 2<sup>3<sup>2</sup></sup> and indeed, if we *were* to do it left-to-right it would actually be *incorrect*!

Of course, as the name suggests, this book makes no pretension to *clear* up school-math in any way. It is just a list, the "notes of the smartest kid in the class". Or, as I would wager, more like the third smartest kid in the class. The first two would definitely not even write down the ridiculous vertical line test!

This book also ends before 9<sup>th</sup> grade (billed as 4-8) and so there is no entry on quadratics, which asides from PEMDAS is probably the biggest confusion in all of school-math. (WEHM, p...)

Here's a very interesting review of "the big fat notebook" in this respect. I could really have not said it better myself. This is what the WEHM manual does! Perhaps I should even change my title to "math for people who hate math" as this layman suggests!

I bought this as an adult, for myself. I know, I know. The thing is, I've decided to pursue a degree and I have to take an accuplacer test. I'd really (really) like to test-out of lower-level math, but it's a struggle since, you know, it's been like 20 years since I divided fractions, solved for x, measured quadrilaterals, or found a square root. Also, I hate math. Always have. Still do, it would seem. It seemed like a good idea to start with basics, so I bought this and a handful of other books so I might have a fighting chance.

Did I mention I hate math? It has always been (and continues to be) the thing with which I struggle most, academically speaking. I find that there aren't really decent resources for people like me, with minds like mine. It's written about by people who "get it", and to them it's so simple that they in turn expect you to just "get it". This book is really no exception. It's more colorful, but it's written with that same hard-and-fast "simple rules to remember" format as math textbooks 20 years ago. Nothing is in plain (enough) language, and very little time is taken to help those of us who can never seem to just "go through the motions" understand the WHY of math. For me, this is the hardest thing. Okay, you flip the fraction. But why? How does it work? Because this goes unsatisfied, I feel like I don't get it, and out of my mind it goes. Lots of folks are this way. I need to see how it works, and then I get that "ah ha!" feeling and it just sticks.

I hoped this book would do that. I hoped it was "Math for people who hate math", but for me, it wasn't. I found the examples to be redundant where they weren't radical, the language to be tricky, and the pace to be too fast, leaving me frequently turning to the internet for supplemental videos.

While it also isn't perfect, I do prefer the Princeton Review's Math Smart book, but the youtube channel Math Antics did more for me than either of these prints combined.

That isn't to say the book isn't without its merit. Only that it's not alternative, at least not in my opinion. It feels like an updated textbook, like what you'd hope elementary and middle schools would provide. But for me, it didn't close any gaps, nor did it make easier those things with which I seem to have such difficulty.

So all in all, if you or your student have a knack for math, this book will accelerate and supplement and is in a lively enough format as to likely not collect dust. But if you're like me and were hoping for something that would offer an alternative approach to understanding math, I can't recommend it.

From here I went straight to "math help" and other books linked to this one on Amazon. I found several such math *dictionaries* that just list the topics and concepts and names, but make no attempt to make better sense of it. They also tend to focus on the lower grades. (I review both Princeton Review's smart book and Math Antics below.)

#### Math-Dictionary-Kids-Guide-Helping

This book is ranked #100 in general kids help and #132 in math books on Amazon.

Again school-math is simply religiously repeated along with the worst offenses and confusions that the WEHM manual actually clears up. Oddly, this dictionary also simply leaves out some very important concepts all together:

There is no vertical line test but also no alternative explanation of what is a function and what is not (WEHM,p...)

PEMDAS is of course repeated with no explanation.

Billed as grades 4-9, there are no quadratics the second biggest confusion in school-math (see above.)

There are 6 separate redundant entries for percent. Some of these entries get into decimals and some refer to proportions but nowhere are the three cases clearly listed. One of the most confounding confusions (because it's actually a practical skill everyone needs) of school-math is to not clearly present the 3 percentage cases and using decimals for these 3 cases as the, by far, most *practical* way of calculating percent. (WEHM, chapter ...)

Now let us take a look at the, inexplicably very sparse, math-help for grades 9-12, starting with the series that dominate this space.

## For Dummies

## Geometry-Dummies #15 in geometry and topology

This book directly fails my *parallel lines and the transversal* litmus test (WEHM, p...) and blindly repeats the whole darn thing. They then even proceed to take school-math's 5 impossible names for 2 angles to a whole new level by applying it to a situation with *two* transversals<sup>4</sup>. This becomes almost comically complicated and they finally suggest covering up one of the two lines so you can even understand their picture (p. 158)

Continuing one of school-math's original sins, the term *congruent* angles<sup>5</sup> (instead of simply saying *equal* angles) are not only fully endorsed, but even hilariously doubled down on with a whole page of deepening confusion. My favorite part is at the end where they proudly state that you will know when an angle is *congruent* when you "figure it out" or when they *tell* you it is *congruent*. Also in typical school-math fashion, nowhere can they bring themselves to say *congruent* angles is just a "fancy" (and incorrect) way to say equal angles. Instead, phrases such as "same numerical measure" and "same degree measure" are thrown around. "Doesn't that just mean equal?" thinks the quaking student. The illustrations are priceless, a great "illustration" of my whole point.

Upon rereading this masterpiece of school-confusion there is even a falsehood. The sides of two congruent angles do *not* necessarily "align perfectly" at all since they could have completely different lengths. This is exactly why *congruent* is the wrong word. So now they have come full circle. First using an incorrect word, and then from the incorrect word concluding precisely the incorrect facts that *would* be true if the word *was* correct.

This, even though it is the *For Dummies* series, very disappointing! Again, there are very few mathhelp books beyond 8<sup>th</sup> grade asides from *for Dummies*.

Remarkable.



<sup>&</sup>lt;sup>4</sup> Once again, a completely new twist on this immortal nonsense. The depth and profundity of the ocean of confusion in school-math is astounding

<sup>&</sup>lt;sup>5</sup> Angles are not shapes, they do not even have fixed lengths, how can they be congruent? It's in every textbook, so certainly not just this book's fault, but they are repeating it.

### Precalculus For Dummies, by Wiley brand

Overall not terribly explained but very brief and probably not helpful when it comes to actually learning a topic. Mostly adheres strictly to school-math, sometimes even phrasing things in an even more formal and confusing manner than necessary for high school-math (*fundamental theorem of algebra*) but never incorrectly. There are even hints of how things might be done better (in the sense of the WEHM manual) but it is not expanded upon or actually worked through (*unit circle.*) These hints remain more of an assertion ("you actually don't need to memorize this") as opposed to instruction how to accomplish that feat. The content is definitely more cliff notes than an actual syllabus.

It is highly doubtful that an average/struggling student would find much solace here. This is more of a primer for those who have already mastered most of pre-calculus and need a refresher.

The typical way school-math does *absolute value inequalities* is repeated (WEHM, p...). This involves flipping the inequality sign *for you* because they are moving the minus sign from one side of the equation *for you* as well. This all happens in one mysterious step that is only rarely explained in any way, certainly not here. If this step *is* explained (*idiots guide*) it is done so graphically and not from the definition of the *absolute value function*. It may not seem like a big deal, but remembered any step where there is no logical explanation, is akin to asking the student to stop thinking and just memorize-the opposite of what they should be learning in math. Of course, it's also extremely challenging and torturous to remember math this way, there are just too many unconnected dots.

The deeper problem though is the lack of acknowledgement that they are indeed missing a step. It just *happens* like so much else in school-math. This leaves many a student doubting their own mathematical sanity, and thus often hating math. Another failure of attempting to actually understand math is another brick in the wall of math trauma.

Here's how the book explains what happens if a quadratic does *not* factor (a so-called *prime quadratic*, WEHM, p…). Such a quadratic *could* have *x-intercepts* but could also *not* have *x-intercepts* we just *don't* know until we use the quadratic formula. When a quadratic has no *x-* intercepts, school-math forces students to suddenly start using *imaginary numbers* for some inexplicable reason, instead of just saying there are *no x*-intercepts. This is ridiculous and reminiscent of the sudden inexplicable introduction of exponents in PEMDAS when multiplying and adding are *first* being understood, except this is much worse because imaginary numbers are a completely separate branch of mathematics that has nothing to do with anything else in school-math, certainly not quadratics (WEHM,p…)<sup>6</sup> This is one of the most serious obstacles most students face when attempting to actually *understand* quadratics rather than just memorizing a pattern to blindly follow. Here the *For Dummies* book conflates the two confusions (prime quadratics, imaginary numbers) into one indecipherable paragraph.

<sup>&</sup>lt;sup>6</sup> Yes, there is the fundamental theorem of algebra of course. But to a vast, vast majority of students that is just a theorem to have heard of, perhaps be able to recite. This is a very advanced notion and has nothing to do with school-math's everyday reality and quadratic graphs.

"The inability to factor means that the equation has solutions that you can't find by using normal techniques. Perhaps the solutions involve of non-perfect squares; they can even be complex involving imaginary numbers"

In fact, upon re-reading this multiple times I finally realized that the phrase "by normal techniques" above apparently does *not* include the quadratic formula (because many quadratics that can't be factored will certainly have solutions when the quadratic formula is used) and thus the phrase "by normal techniques" must simply mean "by factoring". Either that, or the writer is writing so densely and is so occupied with cramming everything into one sentence, that he/she forgot that the quadratic formula *could* give the answer even when a quadratic does not factor. The question what a *prime quadratic* really is, is a central insidious confusion that can lead to serious problems later on. The WEHM manual actually has a whole chapter on this confusion because it is such a deep-rooted issue for many students (p...)

This *For Dummies* entry is certainly confusing, if not incorrect, and shows that this book is actually a collection of more or less, for most students, *indecipherable* cliff notes.

A further example of this is the "explanation" of the fundamental theorem of algebra:

"The fundamental theorem of algebra states: every non-constant polynomial has at least one root in the complex number system...pair up every possible number of positive real roots with every possible number of negative real roots; the remaining number of roots for each situation represents the number of roots that are not real...for now you are mainly trying to be sure you found all the possible places where the curve is either crossing or touching the x axis"

In high school this theorem is usually stated as "if you include complex roots then a polynomial will always have the exact same number of roots as the degree of the polynomial", or more simply "the polynomial will have all its roots". It takes almost a college level of mathematical understanding of the topic to see that "every non-constant polynomial has at least one root in the complex number system" means the exact same thing (it just means we can keep finding roots until we get all the linear factors with roots, so we get all the roots.) Also, no effort is made to clear up the difference between real and complex roots as pertains to the actual graphing of the polynomial. This is definitely geared towards someone to whom that is trivially obvious. I really don't know why this book would qualify for the moniker *For Dummies*.

The explanation of *completing the square* uses  $\left(\frac{b}{2}\right)^2$  as the mysterious secret term and does not even mention the *binomial theorem* which, as previously discussed (p...), is monumentally more important for all of math (including the SAT/ACT) and makes completing the square pretty obvious and easy in that you don't need a secret term that just appears out of nowhere.

Descartes rule of sign and the rational root theorem are discussed at length. I have come across this very rarely in the last 10 years of tutoring for New York's elite prep-schools. I did some research and it seems these rules were more prominent decades ago before the advent of graphing calculators and in college courses now.

The presentation of the *unit circle* is very disjointed, as mentioned in the introduction here. They present the full circle to be memorized as in the worst possible school math approach. Then they mysteriously state without explaining how, that this is all not necessary at all. That is absolutely correct! (WEHM, p...) The reason you don't need to memorize is easily stated as cosine is always x and sine is always y. For some reason they do *not* say that, but go on to make a list of the signs of cosine and sine for the 4 quadrants to be memorized (exactly what can be avoided by understanding what is actually going on.)

In a whiff of fresh air and the only entry I could find that in any way justifies calling this a book *For Dummies*, they recommend *only* doing *completing the square* for the roots when directly told to do so, because it is way more complicated than the *quadratic formula*, which is of course eminently true (WEHM, p...) But of course, when it comes to the *vertex*, they *do* recommend doing *completing the square* again, which is just as ridiculous since we actually have a *vertex formula*! (WEHM, p...) This is once again blindly repeating school-math nonsense.

#### For Dummies Trig

Since trig seems to always be the single biggest catastrophe in all these supposed math-help books it is no surprise that this book is also a complete labyrinth. I'll skip going into the buildup review in any detail. Suffice it to say that it is also once again completely indicative of all these muddled books that honestly seem to be more of a token to be able to say :"well, I tried mom" than anything else, or perhaps given the desert of actually useful information out there some students may manage to squeeze a couple drops of water from this stone.

Functions are not well explained, inverse functions are mentioned with no explanation except for patterns that are presented to be memorized, and so forth and so on....

The unit circle commences with the usual terrifying terminology: standard position (which literally means nothing at all), initial side, terminal side, co-terminal. Then 4 pages the coordinates of the unit circle are presented to be memorized as usual (completely unnecessary, WEHM p...no

As you can see, the x-coordinate here has two values, and the two points are  $\left(-\frac{7}{25},\frac{24}{25}\right)$  and  $\left(-\frac{7}{25},-\frac{24}{25}\right)$ Sticking to rational coordinates You may have noticed, in the last section, that one problem resulted in a coordinate with a radical in it and the other didn't. Radicals can't be avoided (as you read in the news) when doing trig problems, but sometimes you just need to keep things rational. A rational number is a real number that can be written as a fraction. And rational numbers have decimal values that behave --- unlike the decimal values of radicals (irrational numbers). What I have for you here is a way of assuring yourself that you'll get only rational coordinates for a point on the unit circle. To do this, use the following formula, letting m be any rational number:  $(x,y) = \left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$ Nant to see the formula at work? First, I also have a set of the second 1. Replace each m in the formula with 4.  $\left(\frac{1-m^2}{1+m^2},\frac{2m}{1+m^2}\right) = \left(\frac{1-4^2}{1+4^2},\frac{2\cdot 4}{1+4^2}\right)$ 2. Simplify.  $=\left(\frac{1-16}{1+16},\frac{8}{1+16}\right)=\left(-\frac{15}{17},\frac{8}{17}\right)$  $\left(-\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2 = \frac{225}{289} + \frac{64}{289} = \frac{289}{289} = 1$ with -2  $\left(\frac{1-m^2}{1+m^2},\frac{2m}{1+m^2}\right) = \left(\frac{1-\left(-\frac{2}{3}\right)^2}{1+\left(-\frac{2}{3}\right)^2},\frac{2\left(-\frac{2}{3}\right)^2}{1+\left(-\frac{2}{3}\right)^2}\right)$ 

### Pre algebra dummies

### Calculus-Dummies-Math-Science

Just like all the other "for Dummies" books this is again basically just a collection of hard to decipher cliff-notes and it is unclear to me how this helps any "Dummies", or students that are having difficulties understanding school-math. Nonetheless...

One of my main complaints about most high-school calculus courses is the fact that the integral or area under the curve is not introduced until the course is almost over, and then it is not introduced as the fundamental concept it is, but rather mysteriously as the anti-derivative. To this books credit there is no mention of the antiderivative in the introduction. In the introduction the derivative and integral are clearly mentioned as the two most important concepts.

Another surprisingly good aspect of this book is that (finally, WEHM, p...) Xenon's paradox or Achilles and the turtle is used as the first and best (not to mention historical) example of a limit. This is basically as far as I can tell unprecedented and can only be applauded!

The basic review section is also ok, but using a slot machine to explain what a function is is problematic. The book seems to think it is a good example because the output is "mysterious". This is an odd aspect of functions to emphasize, as many functions are not at all mysterious, and this does not seem to me to be a fundamental aspect to understand. The other obvious problem, evident in their picture, is that a slot machine produces several different outputs every spin. Of course, each collection of images produced can be regarded as a unique output, but this is the opposite of clear-it, in fact, requires a further level of comprehension. The slot machine example is not wrong, but it is also definitely not a great example such as a bank account (WEHM, p...). It's just strange that these books always seem to be trying harder to be original than clear.

The explanation of independent/dependent variable is muddled as always (WEHM, p...). They say that "time doesn't cause an object to fall" so time is the independent and distance is the dependent variable, as if a graph of how long it takes to fall a certain distance could not be made. Once again, they seem happily (and incredibly) oblivious to the confusion caused by this in the minds of students. Nowhere do they simply state that, as far as math is concerned, independent variable just means x and dependent variable just means y.

The presentation of the unit circle is an incomprehensible, unmitigated catastrophe as always, topped off with the (also usual) attempt at teacher humor: "circling the enemy with the unit circle" (groan.)

It starts off with the absolutely terrifying terminology of terminal/initial sides and so forth, without ever even bothering to define these useless and unnecessary terms<sup>7</sup>. Next the x and y coordinates of the special points on the unit circle are invoked to explain the values of sine and cosine, again with absolutely no explanation how this has anything to do with the triangles that form the whole definition of trigonometry. In fact, the triangles to the x-axis somehow do not even come up in this chapter on the unit circle until the very end. Then we have two full pages on how to transform radians into

<sup>&</sup>lt;sup>7</sup> It seems a consistent trait of the "for Dummies" books that they know no one is actually reading them to understand anything, and they are literally just going through the motions and *pretending* to explain (p...)

degrees which should actually take, at most, a paragraph. So even this very simple process is made to seem somehow incomprehensible and almost mystically daunting.

Incredibley jumbled paragraph that refres back to pictres incorrectly or confusingly all about size (but then "by the way" concludes that doesn't matter) somehow mixes in the fact that in theunit circle cos is sin is y. usus symemtry arguments for the coordinates instaed

photo

"remember the angle 150 is associated with the point not the angle 30" oh boy, exact oopposite of what needs to be made clear.

They seem to be saying us e x and y to understnd but later....

Jumps through symmetry after symmetry argument for special angles values confusing unneccessay

Special angles and the idea of unit circle inreversible confounded.

Now the triangle is introduced

Ends withmemorize unti circle and all studented take calculus

Limits

Curious statementa" derivative always involves the limit of a functionwith a hole"

Defnitely very dense confusing cliff-notes halfway between learning and memorizing with a penchant for showing off "a secant line is a line that intersects a curve at two pointsthis is abit oversimplfied, but itll do"???? what purpose does that serve?

Gives exampleof derivative before difference qoutient secant turmsinto tangent

Integration

Goes throughleft hand right handmedium approximation sigma notation... before even expalining the beaut8ful block idea never really does just in rieman sums

10 things to rember and to forget great!!

#### For Dummies, Algebra1 #54 in algebra

Equivalent fractions are presented in the worst way ever:

four days a week is 4/7=8/14=16/28 periods of a hockey game 2/3=4/6=6/9 baseball innings 6/9=2/3 hours of a day 23/24=46/48

This is really quiet an amazing example of how subtle absolute school-math confusion can be. It really exemplifies how something seemingly logical and mathematically not even incorrect, can lead to profound lifelong confusion. Each example above starts off promisingly with a meaningful real-life example, but then (remember the whole point here is to explain why certain fractions are equal) 4/7<sup>th</sup> of a week becomes 16/28<sup>th</sup> of... a week. What is this supposed to mean? How does this explain why the fractions are equal?? It *seems* to make sense. Now, imagine the poor 8/9<sup>th</sup> grader trying to find the logic in this, that he/she of course must assume *is* there, because this is math being taught by a teacher or even a school-book, right? But there is absolutely no logic to this! There is absolutely nothing to understand here, because this explains absolutely nothing! Many students of course don't know this yet, although many are already beginning to wise up to the game they perceive as being played here: math doesn't really make sense; they are just pretending it does.

I don't often take it to this extreme, but there is an old saying that the devil is someone who runs through the market square with his fist shut screaming: "I got it! I got it!" making everyone follow him until it turns out there is nothing in his hand at all. This is that.

The completely obvious example to explain the concept of equivalent fractions, that they have for some unknown reason basically taken pains to avoid is:

It takes 2 cups of milk and 1 egg to make 10 pancakes

It takes 4 cups of milk and 2 eggs to make 20 pancakes

The recipe (the ratio or fraction of milk to eggs stays the same)

Or

10 boys and 20 girls, half are boys

25 boys and 50 girls, half are still boys

The only explanation given for equivalent fractions in this book after these nonsensical examples is that multiplying by 5 on top and 5 on bottom is the same as multiplying by 1 which is, of course, not untrue, but technical and only useful once the basic premise has been grasped.

While explaining the need for a common denominator when adding/subtracting fractions this book does say quarts and gallons is like 1/5 and 1/7, and so we can't add them yet. They never really explicitly say you can't add them because they are not the same size, but close enough. Although I do think pictures of pizza slices that are not the same size are far more obvious. But at least they

tried and got close, and most importantly didn't make up weird cross-multiplication tricks (p...) to *obscure* the meaning.

For some reason the powers rules are all well explained except for the most confusing one. Negative exponents are just stated for blind memorization. Of course, there *is* a simple explanation readily available (WEHM, p...)

For order of operations they just opt out of even mentioning PEMDAS at all, a sentiment I can fully appreciate (WEHM, p...). The problem here once again (p...), however, is that by simply not mentioning something that the students do have to deal with and are still confused by, in school-math reality, the book is not really helping very much. Students may be left wondering if this is the same thing as PEMDAS then why did we do PEMDAS if we didn't need it. None of this is addressed of course. Just as in so many other books reviewed here they are more or less admitting school-math is flawed without saying so, and then pulling their punches (p....).

This book doesn't even mention parentheses initially. Instead, they are not brought up until after the order of operations has been established. This is fine, in fact it could even be argued that this approach is more precise as parentheses themselves are not strictly part of the order of operations but rather supersede it. But, Oh Boy! What about the fact that PEMDAS starts with a P? That has to confuse the many students who diligently memorized PEMDAS and are desperately clinging to it, especially since they make no effort to reconcile the two approaches.

So, they start with the 3 levels of operations:

## powers/exponents multiplication/division addition/subtraction

And then they make a mathematical mistake, and a grave one at that! Just as in *the big fat math notebook* (p...), they now *incorrectly* state that the three levels in themselves are to be performed from left-to-right! Wouldn't that be grand! It would make the dreadful, anti-mathematical left-to-right rule (now in 9<sup>th</sup> grade by the way no longer relevant, WEHM, p...) ring mathematically valid. There is only one small problem with this brilliant plan, it's not true. Exponents are *not* performed left-to-right!

Consider one of the two basic power rules  $(2^3)^2 = 2^6 = 64$  and what would happen if we discarded the parentheses  $2^{3^2}$ . According to these two books the glorious left-to-right rule tells us to do  $2^{3^2}$  leftto-right anyway so we would get the same answer as with parentheses  $8^2 = 64$  which is *incorrect*. The correct answer is  $2^9 = 512$  as exponents always apply to the thing they are directly above (WEHM, p...)!

Once again there is no real-world example for the distributive property, even though this is probably one of the few cases where the "math is all around you" mantra is, in fact, eminently necessary (WEHM, p...).

When the all-important, often neglected, binomial theorems are presented, the minus case is not made for some reason. When the essential difference of squares is presented, the book includes the cubic version as if that were anywhere close in significance, which it absolutely is not. So, this essential tool is buried here amongst useless rubbish as always.

The grouping method for quadratics never happens which is of course very odd (but by no means rare in these supposed math-help books, p...,p...,p...) considering that is what the majority of school-math time (at least the part about quadratics) from 9-10<sup>th</sup> grade is mostly devoted to (WEHM, p...).

Grouping is presented as part of normal factoring with no connection to quadratics for expressions with up to 6 terms. This is completely useless except for some very rare instances and of course misleading, because this *coincidence* is in no way a fundamental mathematical concept as normal factoring is. Once again school-math is busy burying important ideas along with useless or way to specific cases. Then in a crowning achievement of confusion when grouping fails for these absurd expressions, they are referred to as "prime in the algebraic sense" which is at best a term borrowed from group theory and actually just terribly confusing for and wrong (WEHM, p...)

Finally, there *is* a sign that the book understand that grouping quadratics is the main occupation of 9-10<sup>th</sup> graders. But, oddly (but not uncommonly), grouping quadratics itself is *not* at all explained (I guess they assume students have already memorized it somewhere else.) Instead the different signs of a and c are presented as separate cases to be memorized, ostensibly to simplify the grouping process which they have neglected to explain as it pertains to quadratics at all.

Of course, actually helpful pieces of advice such as checking if the discriminant is a perfect square to see if factoring is even possible (WEHM, p...) is not mentioned at all.

#### At first glance this seems to be my book! Consider, for instance, a quote from the introductory pages

"... ideas are center stage with the focus not on common core math, but on student thinking. Teachers work every day to help students improve their thinking and to provide students with new ideas when they need them and when they're ready for them...Students' ideas are an important beginning place for math learning rather than being seen as an irrelevant distraction.

Many people in this country have experiences with school math that can be summarized as *rules without reasons*. They were told to do this in situation A, but do that in situation B. They never understood why and they struggled to remember whether to do this or that in situation A. And they struggle to tell situation A from situation B so they just applied what they hoped was the right rule in the right situation and prayed that they could earn enough partial credit to pass the test."

This is what my manual does! This is it! Can I use this? It would be perfect!

The *unbelievable* chaos caused by the left-to-right rule (WEHM,p...) as evidenced in the latest internet flurry that even reached the New York Times as mentioned in the introduction here. This book does not question the all-mighty acronym PEMDAS or the left-to-right rule although this is really where everybody gets so completely bamboozled that that they still are confused epochs later.

Plenty other instances abound where this book, despite its motivational promise, does little more than simply repeat school-math.

This goal of this book is to help make the common core less daunting and to help parents explain math to their kids. This is achieved through a mixture of inspirational speech about why the common core is not so bad after all, and then explaining the Common Core approach to different topics. These approaches purportedly have more to do with thinking than previous iterations of school-math (this author sees virtually no difference, except in *lip service* paid to the concept of understanding rather than memorizing.):

Also, the backbone of school-math is in my opinion, as previously mentioned: lines, quadratics and powers. PEMDAS and quadratics are in turn the two worst cases of mathematical malpractice and greatest singular causes of math trauma in all of school-math. Even though the Common Core does indeed include quadratics (irresponsibly adding imaginary numbers into the mix at this very inappropriate moment, WEHM, p....), and even though quadratics are probably the single biggest problem in school-math for the reasons above, they are nowhere to be found in this book.

## Idiots guide

## The complete idiots guide-precalc

#1171 calculus #181 mathematics reference #435 popular &elementary pre-calculus

This amorphous series follows a "standard pre-calculus curriculum" for both high-school and college, whatever that means. It would seem those are two very different things, especially depending on where you go to college. Written by an awarded high-school teacher, one would assume it would be clear that having a bunch of material not at all relevant to high-school math would confuse both idiots and complete idiots alike.

The unit circle is memorized (unnecessarily, WEHM, p...). Then there are a mere 3 very compressed pages of explanation of the whole thing. Next there is a chapter entitled *oblique* angles. I could not quite figure out what it was supposed to be about. As the name implies it was, at first, about non-right-angle triangles. Soon however the chapter veered into talking about quadrants and reference angles, in other words back to the unit circle, which is however *based* on *right* triangles. Then suddenly the chapter was about the sine and cosine rule, so now once again actual non-right angle (or oblique) triangles. Very confusing.

Absolute value inequalities are handled in an extraordinary manner even for school-math. It is suggested that the student use two distinct methods, one for greater than inequalities and another completely different one for less than inequalities. Both of these methods make no reference to the definition of absolute value, which is a recurring school-math issue, and skip the step where the sign mysteriously changes sides and flips the inequality (p...). In effect, this is asking students to blindly memorize. The first case or method leads to a double inequality. It is not explained what to do if the expression that now must be manipulated in the middle of the double inequality has a negative sign, confusing.

In contrast to McKellar (p...) this book does not even mention the terms conjunction and disjunction.

Even though functions are arguably the most important concept to thoroughly understand for calculus, there is no explanation of functions to be found here, not even a refresher.

The quadratic formula is disparagingly referred to as a plug and chug process. This line of thinking is to be found everywhere in math-help books, which is odd since in school-math *reality* the quadratic formula is in fact the panacea for a troubled student when it comes to quadratics (p....) Not to mention that it is one of the most amazing results of early mathematics. It has just recently become fashionable to reject it as "just a formula" which leaves one wondering what these people think the point of mathematics is, and what they would say about  $E = mc^2$ , Maxwell's field equations or virtually anything else in science.

Once again, in a similar vein, completing the square is given far too much importance (p...) this is justified by saying this will be useful for conic sections, a pretty ridiculous idea in the harsh cold reality of school-math (p...)

To the books credit it does show how to get quadratic formula from completing the square, although I don't know many idiots or complete idiots who would care. The name is a misnomer, this book is for students who are very good at algebra and just need a refresher. Once again cliff notes.

Then the book launches into a very detailed account of the fundamental theorem of algebra and 5 pages on Descartes rule and the rational root theorem, of which the fundamental theorem of algebra is the only one usually touched on in high school.

The fact that this book is arguably more aimed at the proficient college student is underscored by an extensive treatment of matrices that will never be seen in high school classrooms. This includes calculating determinants, minors, cofactors, matrix manipulation, expanding row sand columns Cramer's rule and row echelon form!

## The idiots guide algebra by Alpha

### #1379 in algebra

Even though these are cliff-notes, time is taken to introduce functions in the usual roundabout formal manner, through the abstract definition of a relation. After this early episode, relations never come up again. This confusing and useless approach is for some reason the consensus in school-math education (p...). Then we have 3 pages about ordered pairs switching around to explain what a relation is:

"many times, there is a rule that explains the pairing we have in a relation or a function, but not every time"

And finally, much later, after way too much of this, instead of simply saying you can't have two different blood pressures (or amounts of money in one account) at the same time, the vertical line test (WEHM, p...) is rephrased as: you can't have two outputs stacked over each other. Again, there is no explanation why this is true, or why this is in any way important to functions.

Point slope form is simply unapologetically presented (WEHM, p...)

Absolute value inequalities are handled in the same confusing way as in the pre-calculus edition but here there are no worked examples.

### Quadratic factoring

The book goes through the different cases (a=1, a>1) even randomly once mentioning the fact that prime a values cases are easier to do without additional tricks. Then, however, the author picks one easy a is prime as a worked example but does not give a non-prime example where grouping would be necessary. In fact, grouping is not mentioned here at all except for the extremely rare, variable only, cases with 4 terms that are not even quadratics (once again, p...). This in spite of the fact that grouping quadratics is of course what school-math spends up to 3 years doing (p...) As usual, there is no summary quadratic factoring table.

### Check why mckellar says rational is the trick?

Check they all use secret term idiots guide yes, number of pages conjuntin disjinction mckellar

## **Demystified**

## Algebra demystified Mcgraw hill

The motto is: "hard stuff made easy" but in fact this is just another book of cliff-notes/knitting patterns. This is probably the *best* collection of knitting patterns I reviewed, everything I saw was actually correct (pretty rare), but there is very little explanation of anything. Why this is called demystified is in itself a mystery.

Once again adding fractions is not explained in a sensible way. They divide pizzas into 12<sup>ths</sup> but never say the magic words: pizza slices must be the same size to be counted (p..., p..., p...)

Percentages *are* done with decimals and equations and there is no mention of the proportion approach which I can only commend. But then, for some reason, the three distinct cases are not clearly presented as such, although there are worked examples of each case. It is just never mentioned that there are indeed three cases.

Also, percent increase/decrease is done by adding/subtracting said percent after calculating it separately. This is fine for a start, but then there is no mention of the fact that the more practical way of doing a 20% discount is actually to just take 80%, or that a 25% increase is just 125% or multiplication by 1.25. Again, this is odd because percents are indeed one of the few actually necessary life skills in math.

Factoring to find roots is well introduced theoretically, but the actual factoring is done in a completely separate section of the book and even there the a=1, a>1, a is prime (WEHM, p...) cases are not clearly delineated or explained. Even a=1 is not explained at all. It just starts happening without even stating of how the pattern works, much less an explanation of what's going on. The a>1 cases are done by trial and error (even for non-prime large a's) with absolutely no explanation, and there is no mention of grouping in this context (once again in reality a>1, grouping is what school-math spends 3 years doing.) In another separate chapter expressions with 4 terms are subjected to grouping. This time, however, these expressions are the *middle* step of grouping as done in school. Unlike many other help-books that do grouping only for variables (p...) they are using an expanded quadratic here but, for some incomprehensible reason, neglect mentioning how you get to this point. There is no indication to the reader how this ties into either factoring a quadratic for a>1(grouping) or, for that matter, roots. It seems they are assuming the student already knows how to factor and they are merely supplying exercises and *intermediate* steps, which is odd.

In a radical shift from most other help manuals that go into excessive detail about completing the square (p...), here it is just mentioned by name once, but never even executed a single time, which is obviously also not a good outcome for students.

### precalc

Here we have once again functions being defined as a special kind of relation (p...), and quite a mess is made of it:

### photo of definition mess

This supposed math-help book actually tells the student to complete the square to find the vertex, which is of course completely unnecessary (WEHM, p...), but is a standard school-math confusion.

The chapter "graph of a quadratic" is, for some reason (because it's easier?) only about vertex form. How to get the graph from the other forms is not mentioned here, possibly because the idea is to (unnecessarily) always first complete the square?

In the trigonometry section, this book does say sine is y and cosine is x, which is indeed the key to not memorizing the obscenely intricate unit circle (WEHM, p...), but then oddly no conclusion (such as the signs of the trig functions in the different quadrants) is drawn from this. Also, no explanation is offered for this important fact, such as the actual definitions of cosine and sine. Really shorthand cliff-notes and knitting patterns.

In another bow to school-math, synthetic division is presented with no irony as if it was an improvement on polynomial division which it decidedly isn't (WEHM, p...). Synthetic division is just another random knitting pattern that only applies in limited cases and obscures what is actually going on-division.

In a refreshing twist, imaginary roots of quadratics are finally done separately and this time *not* intermingled with the understanding of what a quadratic actually looks like! (p....)

Also, in another move untypical for school-math, log and ln are finally treated as the same topic, which they decidedly are!

## Danica McKellar

## Penguin publishing

These are actually by far the best books I found and they do include at least *quadratics* from 9<sup>th</sup> grade. The reasoning is down to earth, the author really understands what's important and guides the student there very gently. Many of the usual pitfalls of school-math are avoided, these books do almost no wrong! But unfortunately, they often also do nothing at all.

This very different and generally well executed grade school book is a bit of a concoction. I can't make out how the topics are chosen, by personal favorite or following the Common Core.

A lack of actual teaching experience is evident throughout. Most things that *are* covered are more or less well explained (*functions* unfortunately is a terrible exception.) But most of the tangled webs and "mathematical booby traps"<sup>8</sup> that students spend so much of their time trapped in as they attempt to navigate school-math are simply *ignored* (for example, *parallel lines and the transversal* or the *exponents* in PEMDAS, WEHM, p..., p...) This is an option that students obviously don't have. The fact that students must be able to follow and mimic what teachers and textbooks are doing, and not just comprehend the material in its purest form (Alas!) is not taken very seriously. There are no summary tables where all important school-math tricks are listed together. *Quadratic grouping*, for instance, is somewhere completely different than *completing the square* and the *quadratic formula*. There is no flowchart overview of all of *factoring*, that would address helpful detailed points such as factoring when *a* or *c* or both are prime, or at least present the whole thing in one place as in the WEHM manual, p...

*Completing the square* is 14 pages long for some reason (see above), while the *whole* of quadratic factoring is also *just* 14 pages. The *quadratic formula* is called a "last resort", but in school-math reality, when finally revealed, the *quadratic formula* is often the miracle cure for years of endless, meaningless *factoring* and *completing the square*. This whole set up of the book *Hot x* does not reflect the reality that in school *factoring* takes up 3 years while *completing the square* is at best 3 months.

The overall picture I get is of a well-intentioned, very capable author who really is doing things much better than the usual course of school-math, but who has not been fully immersed in student's everyday school reality. This is not a teacher with real world experience. If this excellent author had more experience teaching, I'm sure:

@adding subtracting fractions vs multiplying fractions (i.e. when to form a common denominator) would be presented more clearly as the result of needing same size slices to be able to count them (add.)

@the distributive law would have a real life example.

@percentages would be written up as a table with 3 cases using decimals.

@the sausage factory for functions would be changed to something else (see reasons below) and the vertical line test would then not just be repeated with no explanation.

@parallel lines and the transversal would be mentioned and clarified.

<sup>&</sup>lt;sup>8</sup> (WEHM,p...)

<sup>(2)</sup> There would be a table/flowchart (as in the WEHM manual) for the whole of quadratic factoring, what it means when quadratics *don't* factor would be explained and the incredible confusion caused by involving imaginary numbers in the first explanation of quadratics would be remedied.

@Dependent/independent variable (WEHM,p..) would be addressed,

@conjunction disjunction would not take up...pages since this is really not that big a deal in school (although some teachers do like to harp on it) and mostly just sounds scary and confusing and there is a very easy way of explaining it without those terms (WEHM, p...) This is similar to explaining completing the square in even more detail than school-math, totally unnecessary and useless for the vast majority of students.

But there also many things these books get refreshingly correct:

It is pointed out that PEMDAS could just as well be PEDMAS etc.! (WEHM, p...)

The slope-point formula is derided as more complicated than finding b by plugging in a point! (WEHM, p...)

There is a whole separate chapter for the all important binomial theorems that are buried under mountains of useless factoring in school! (WEHM, p...)

The definition of slope and rise over run is executed perfectly!

At one point she warns the reader that one must be very careful about equating human language to math, this is also one of my complaints! (WEHM, p...)

She calls math "gym for the brain" (as opposed to the usual "math is all around you" mumbo jumbo) this happens to be almost verbatim from my introduction! (WEHM, p...)

Of course the clearest difference between these books and the WEHM manual is simply stated the attitude. While these books sometimes even seem to be justifying school-math labyrinths such as completing the square, it is all done in a cheerful carefree tone. There is no hint of the idea that school-math may in fact be "abusive" and the students the victims. There is no hint that everyone may hate school-math for some, it turns out, pretty darn good reasons. Even when this series points out that PEMDAS could be PEDMAS and then actually goes on to completely ignore the exponents part of PEMDAS (because it is ridiculous that 6<sup>th</sup> graders should suddenly have to deal with exponents while learning the order of addition/subtraction and multiplication/division and parentheses, WEHM, p…) these books never once unapologetically take the side of the downtrodden math student who, of course, does have to deal with exponents in PEMDAS and all the other school-math pitfalls that the series chooses to comfortably ignore. Just like Khan Academy these otherwise fine books pull their punches.

Here are those general points broken down book by book.

#### Kiss-My-Math

The distributive property is explained using the example of someone saying hello to everyone at a party and when certain people stick together more than others (multiplication) then you only say hello once. This is obviously again just a silly mnemonic device and not an actual explanation of the rules of the marketplace where math in actual fact comes from (WEHM, p...). Understanding the difference between addition and multiplication is fundamental, understanding these basic properties as facts of the marketplace and not random silly constructs (made up to torture students no doubt) is even more so!

The awful example of a sausage factory is again used for functions, see *Hot X* below.

This book has a lot of intersection with *Hot X* so most of the topics are treated there.

### Math-Doesnt-Suck

#62 mathematics teaching and study #25 stem education #121 math teaching materials

#### Grades 4-6

*Percentages* conversions to decimals, decimals to percent are covered. Multiplying a value by the decimal version of a percentage to *get* a percentage of said value *is* covered. The *other* two cases for percentages are *not* mentioned as far as I can tell I (see below for other cases), and I did go through all percentage entries in the index. One of the most *confounding* confusions of school-math (because it's actually a *practical* skill everyone needs) is to not clearly present the 3 percentage cases and using decimals for these 3 cases as the, by far, most *practical* way of calculating *percent*. (WEHM, chapter ...)

This despite the common core guidance for 6<sup>th</sup> grade, listing the 2 more advanced cases, below. (The first case of simply finding what percent a number is of another number, must be previously listed in 5<sup>th</sup> grade):

Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent -common core standards 6<sup>th</sup> grade

The addition of fractions is covered using pictures of pizza but oddly there is no direct mention of the size of the slices to explain why we need a common denominator to add/subtract fractions (p...). Instead students are taught the mnemonic device of saying the fractions must agree "deep down" which is somehow connected to morals and the bottom of a fraction.

When adding/subtracting fractions the denominators must be the *same* size as you cannot count pieces of pizza unless they are the *same* size. School-math hardly emphasizes this actual *explanation* of *why* denominators must be the same. Instead students are fed silly memorization drills such as: the fractions must *like* each other, so they must be *like* fractions. On the other hand, when multiplying fractions, you do *not* need to make the denominators the same. Far too many students can never *quite* remember which one, addition or multiplication, needs the common denominator (why don't multiplying fractions *like* each other?) I am forever explaining the *why* of this

this to 11<sup>th</sup> grade SAT students. A shocking number of them never really understood this at all, they just memorized it and are consequently constantly forgetting it.

Switching from "liking each other" to "deep down" won't help these future 11<sup>th</sup> graders remember if its for addition or multiplication either. Oddly, in the next book, *Hot X*, which covers the higher grades, where adding/subtracting fractions should already be clear, a quick one sentence revision *does* mention the size of the slices in passing.

"Pandas eat mustard...." or something is used instead of "Please Excuse My Dear Aunt Sally" which does not help anybody. But, to the books eternal credit, it does admit, even clear up, the fact that PEMDAS could just as well be PEDMAS etc. (WEHM, p...) This is an important admission of a ridiculous truth that most school treatises avoid like the pest.

This book does not warn of the impending disappearance of the horrible left-to-right rule. Exponents are simply not addressed at all in this book at all, even though they are still part of the acronym PEMDAS, despite switching to pandas. Students in school still have to do exercises *with* exponents of course.

The *distributive law* is not in the index (or the book as far as I can tell) at all even though it is essential and needs to be well introduced in grades 1-6 (WEHM, p...)

Cross-multiplying is introduced as a trick to determine which fraction is greater. This is a neat trick I like to use for standardized exam prep especially. There is nothing wrong with it per say, but there are 2 other uses of cross-multiplication that are far more common in school and far more common amongst students. These other uses of cross-multiplying actually represent a grave danger of confusion if not properly used (WEHM, p...).

The table of contents not about math. The index more or less works it seems though.

Math is extoled as Gym for brain no "math is all around you" mumbo jumbo here! (WEHM, p...)

#### hot-x-algebra-exposed

#111 math study and teaching #20 teen and adult algebra #50 stem education

Although I have really been enjoying this series for all the reasons listed above even a routine examination of the mathematical content and explanations of concepts reveals some pretty terrifying pitfalls.

I'll start with the good old question "*what is a function*?" this is one of the most important questions to correctly answer if a student is ever to have a chance to really understand the rest of math.

In the WEHM manual (p...) a *function* is a bank machine (ATM) that tells you how much *money* is in your account at a given *time*.

In this book, a *function* is a sausage factory and it produces *sausages* from *ingredients*.

As much as I really think this is a book I can for the most part recommend, this explanation of functions is one of the *wurst* I've ever encountered.

It leads to some pretty obvious problems such as don't sausages mostly have multiple ingredients and much worse how can two *different* ingredients make the *same* sausage? I just don't see taking chicken and getting the same sausage as when I used beef. But I can put my ATM card in a bank at different times and have the same amount of money. In fact, how would the sausage factory work that always produces the same sausage (i.e. the account balance that doesn't change). No matter what ingredient, I would always get the same sausage-I'd definitely doubt if this was even really a sausage factory. On the other hand, since sausages are mostly made up of multiple ingredients it seems no stretch in an 8<sup>th</sup> graders mind (just being introduced this way to functions) that one ingredient could lead to several different sausages. That actually makes more sense than different ingredients leading to the same sausage. Groan. This is exactly the opposite of what needs to be made very clear at this point in a child's mathematical development (WEHM,p...): you *can't* have two different amounts of money at the same time but you *can* have the same amount at two different times.

Given all this this, the following quote from the function chapter could really have quite a catastrophic impact on an 8<sup>th</sup> grade mind trying to decide whether to hate math or not:

"an input value (ingredient) can't be paired with two different output values (sausages). On the other hand, more than one ingredient is allowed to result in the same sausage... each ingredient knows where it's going... factories should be trustworthy and dependable"

This chapter culminates in a blind repetition of the *vertical line test* (WEHM, p...) one of my litmus tests like *parallel lines* (WEHM, p...) followed by an inspirational quote.

*PEMDAS* is presented but exponents are never used in any of the exercises, a luxury students don't have. To the books eternal credit, it does point out that PEMDAS could just as well be PEDMAS.

*Slope-point formula* is derided as more complicated than simply finding b which is something I always emphasize to my students (WEHM, p...) However, it's not just that point-slope form is inconvenient, especially if the student must put the line back in function form The real reason *not* using point-slope is important is because it's the very first instance of finding an unknown from a piece of information, using a point to find a variable. This fundamental concept comes up again and again in mathematics from quadratics to calculus and forever on. This is not explained.

The monumentally important and very much underrepresented (in school-math) *binomial theorems* are a given their own whole chapter!

Rise over run is done very well.

On p.57 the *size of pizza* slices is finally referred to as the explanation of the common denominator for adding/subtracting fractions. Unfortunately its just one line of review in this more advanced book. As previously stated here (p...) this explanation is never directly used in the earlier books where it is really important (p...) Too little, too late.

*Conjunction/disjunction* takes up a full 3 pages even though these terms are at best secondary or tertiary in importance to the students (as is evidence by their absence in most other math-help books). The reason no one needs them is that they really make a relatively simple concept very confusing. There are a few teachers and books that like to make hay of them, but this is generally avoided, and even if they are to be explained it should be done the other way around. In other words, explain the relatively simple concept (what is really going on) first, and *then* say we call this *conjunction* and this *disjunction*, instead of starting from these alienating terms that admittedly *are* important in more advance math, but not here. This is another example of McKellar being good at math but having little experience actually teaching, it would seem.

The treatment of *absolute value inequalities* avoids the whole issue (see For Dummies review, p..., WEHM, p...) by *testing* values which is, seemingly, a practical way of avoiding confusion here. Alas, in school the textbooks, teachers and exercises all go through the mysterious contortions explained in the other review (p...) and so if a student were to simply follow this book, they would be completely confused about what is happening in school. Of course, another problem is that testing values doesn't work for algebra and variables, it only works for numerical values that can be tested.

Also, when doing absolute value with *equalities* she puts the minus sign on the side of the answer in the two cases, which is strictly speaking incorrect (although in the case of equality it makes no difference.) This is precisely the "error" that leads to so much confusion when we get to inequalities (because there moving the minus sign from one side of the equation to the other flips the inequality)

My litmus test, *the transversal and parallel lines* (WEHM, p...), is simply not mentioned anywhere. I can understand the sentiment. The whole thing is indeed completely trivial mathematically and deserves never to be mentioned again, but unfortunately students don't have the luxury of ignoring what they don't like or understand in school, so this leaves them in the lurch.

As discussed in the introduction to the comparative analysis (p...) the McKellar series books are amongst the few math-help books that actually cross the line into 9<sup>th</sup> grade. So, this is a book written in the manner of a 1-8 grade user-friendly help guide that, quite distinguishingly, actually touches on *quadratics*. But, while dipping into a part of the second huge confusion of school-math after PEMDAS, namely quadratics (and their factoring or not), there is little to no explanation or warning for what is about to happen to students.

For instance, at several points in the book it is asserted without explanation that factoring only works for *roots* that are *rational* and also that *if* the roots are rational it will factor. The second part of this statement is actually, believe it or not, a result of *group theory* as explained in the WEHM manual (p...) and by no means self-evident. Even the first part of the statement is not something your average 9<sup>th</sup> grader will immediately understand, if at all, certainly not without any explanation as is the case here. The real question is why is this being mentioned at all. ?????

This book has the 14 unnecessary pages of *completing the square* as mentioned in the introduction to the whole series here (p...).

Also, what happens or what it means if a quadratic does *not* factor (so called *prime quadratic*, WEHM, p...) is never touched upon at all (see *For Dummies* review, p...). This can all be very confusing both immediately and long-term.

The ambush on the students in the form of the sudden, unwarranted inclusion of *imaginary numbers* in the quadratic formulas (WEHM, p...) is just never mentioned or addressed even though the quadratic formula is mentioned. Just as in the case of *parallel lines and the transversal* this might be

mathematically absolutely reasonable but just does not help the students navigate the treacherous shoals of what is thrown at them in school-math.

For some reason the *quadratic formula* is referred to as the last resort rather than the panacea it actually is in school-math reality. This is no doubt a result of the same thinking as explained in the "a word about completing the square" (p..) section in this manuscript

And finally, as pointed out in the introduction to this McKellar section (p...) there is no user-friendly *factoring table* that sums up the whole 8-10<sup>th</sup> grade factoring labyrinth as there is in the WEHM manual (p...), there is no mention of cases where either a or c is prime that make the whole thing faster and easier (as there is in the *idiots* guide and the *WEHM manual* of course, for instance).

There is no mention *standard/general* form terminology conflict (WEHM, p...)

The silly *distributive property* party analogy is used here again (p...)

The ridiculous confusion surrounding *dependent/independent variable* is just not mentioned, again a luxury suffering students simply do not have.

The other two books by McKellar I assumed were in a similar vein.

Not-Open-This-Math-Book

Girls-Get-Curves-Geometry-

## **Barnes and Nobles**

I went to the Barnes and Nobles on Broadway and 81 in NYC and took a look at their study help section.

Asides from the some of the books discussed above, this is what I found that was evenly distantly relevant.

### Schaum's Precalculus

A highly condensed book of exercises with a very brief salient explanation at the beginning of each topic. Salient to the point of mathematical rigor. This is a good collection of worked out problems. But there is little help here for a student who doesn't already basically understand. The factoring of a quadratic to find its roots, for instance, is not even mentioned in the introduction to quadratics. Finding the roots is finally found in an exercise that has the introduction to the quadratic formula and the discriminant as its answer. Actually, the reader is referred to the section *linear and non-linear equations*. Here in turn the entirety of solving quadratic equations is contained in a fourth of a page. Cliff-notes.

#### Algebra for the Clueless

The distributive law is presented with just variables with absolutely no explanation.

When explaining quadratic factoring this book forgets to explain sum and product trick (for a=1) at all launches into 2 page discussion about signs of the numbers instead.

In another variation on school-math, trial and error is used instead of grouping for x > 1 quadratic factoring cases. Then later, grouping is introduced but just for the very rare cases with only variables that happen to respond to grouping (not quadratics.) For some reason this takes up 2 pages while the (far more necessary for school-math success) completing the square is disposed of in one meagre sentence.

Cliff notes.

### Basic Math and Pre-Algebra homework helper

This book ends at multiplying polynomials so it is puzzling that it says pre-calculus in the title.

Not only is the fact that adding fractions requires the same denominator just stated and not explained in any way at all (see p...), but it then stated that the student should "remember the golden rule of fractions, what you do to the top you must do to the bottom" The next sentence does kind of repeat the statement using the word *factor*, but 20 years of experience have taught me that one of the most common confusions is students not understanding that this so called golden rule is only is true for multiplication/division. This is not really made clear at all. The problem is of course that this so called golden rule *is* true for equations, but *not* true for fractions (except when it comes to multiplication/division.) This feeds straight into another common confusion (fraction rules vs equation rules). Catastrophic.

There is a chapter called money problems and it is not about interest. As far as I know this is not a math topic.

Cross-multiplying (p...) is offered up as the way to solve proportions but the actual algebra that lies behind the memorized pattern of the cross (which leads to students never really understanding algebra, WEHM, p.), is compressed into one line where the two crucial steps happen simultaneously. A recipe for disaster.

After the usual method of using proportions for percentage problems is expounded on at length (as in school-math), when decimals are finally used for percent it is phrased as:

"the whole times the percent is equal to the part times 100"

Which is just about the most unappealing way to phrase this life saving insight (WEHM,p...)

Then the examples for this (using equations instead of proportions for decimals) has examples for the two simpler cases (finding a percent and finding a percent of) where equations are still better than proportions but only slightly advantageous, and then no examples for the hardest case (17 is 23% of what?) where equations really, really help make things easier.

#### Must-know pre-calculus

Their motto is "where other books ask you to memorize, we will show you the must know"

*Functions* are not explained in a common sense way. As always in school-math (McKellar being the exception, but rather unfortunately), they start with *relations*, a (from the student's point of view) mysterious thing that doesn't come up again, unless you study math. For some reason it is deemed necessary here at the very beginning of school-math in 8th grade.

"functions are relations that guarantee for each legitimate input value there will be exactly one output value"

The word legitimate is a reference to values not in the domain, bringing this up here in this "definition" is obviously confusing. Of course, as always, no one expects students to actually think about this anyway in any meaningful manner. This "formal" definition of a function is just brought up in a pretentious attempt at mathematical rigor. It's basically vanity on the part of these authors/school-math.

Then 2 pages later we are already plunging into super complicated compositions etc.

Once again this is just a bad collection of cliff notes.

A chapter bearing the name Quadratic equations starts with the surprising and confusing statement that both real and imaginary numbers can be graphed on a number plane! Not untrue of course (2 different number systems or maybe referring to the entire complex plane?), but by not explaining the difference this becomes a profoundly deceptive statement, especially in conjunction with the rampant confusion caused by including complex solutions when first introducing the quadratic formula (so can the imaginary roots of a quadratic be seen in the graph or not, asks the bewildered student?)

The book skips all of quadratic factoring, and then suddenly some very remote complex numbers stuff is covered at length, like the parallelogram method for adding complex numbers. This is not a regular part of any high school syllabus I'm familiar with. Why this is suddenly covered here is not clear, or who this is aimed at. College students?

Then the book goes right back to graphing quadratics with no further mention or explanation of what complex number have to do with the graph. This chapter is nominally about quadratics but then abruptly ends, having never explained the connection between quadratics and complex numbers at all, either in terms of the graph or how we get complex roots from the quadratic formula even.

Incredibly bad.

#### Step by step lessons and practice for algebra 1 "U CAN" from the makers of For Dummies

These are once gain cliff notes but this book is also trying to be funny. One chapter is called *working with numbers in their prime* and so on.

Adding and subtracting fractions and the common denominator (p...) is not explained. Not only that but the book suggests using a (now 4<sup>th</sup>, WEHM, p...) cross-multiplying technique instead of understanding. It actually says here verbatim "your teacher was wrong you *can* add fractions with different denominators", meaning using this cross-multiplying variation of course, but still this is enough to permanently confuse many. Then the trick is expanded mindlessly to 3 fractions.

This is all billed as some revolutionary new way of doing fraction addition (a typical instance of supposedly revolutionary changes being much worse than school-math itself) Then the next chapter says this is how you had to do it before, and goes thru all the least common multiple stuff as if it was something completely different, which of course it absolutely isn't. The only difference is skipping all important steps and performing the cross-multiplying trick, and whether you multiply to get the common denominator or whether there is a smaller denominator that could work as well.

There is even a section called "10 math demons that trip people up", which indeed list adding fractions but then just does not explain anything any better it just shows the algorithm again.

There is a long rendition of the story about the pope and Michelangelo and the painting of the chapel in Rome where the pope asks "when will it be finished?" and Michelangelo responds "when it's done". This is somehow supposed to be about how long factoring can take. The incredible irony here (seemingly lost on the author) is the fact that factoring, of course, is sometimes never done, because not all quadratics can be factored in the first place. This is often a deep seated confusion, even among more advanced students (*prime quadratics*, WEHM, p...).

The chapter on quadratics is called trinomials. A lot of things can have 3 terms not just quadratics (WEHM,p..) This is common use of the term in school-math, but it is nonetheless confusing.

Factoring is extremely condensed to the point of being illegible, but there is apparently enough room for a lot of banter about popes etc. and even a big side box about *Gauss* the child prodigy.

Trial and error is the only method offered for quadratic factoring a>1 cases.

Once again grouping is only used for purely variable expressions not quadratics. Come to think of it, these variable expressions have 4 terms so why are they in the chapter called trinomials...groan!

In an unexpected move this book actually mentions one of my favorite facts to drive home how math is just common sense. Why do we operate in base 10? Why do we have 10 symbols? Because we have 10 fingers! Amazing.

Percent

percent to decimals/percent to fractions/fractions to percent

Oddly, the 3 types of percent problems are finally listed in a table together in this book and even performed with decimals! (WEHM, p...) I believe that makes this book unique amongst all the books I review here! The only problem is finding a percent of something is done like this:

20% of 41 = 2 times 4.1 Instead of 20% of 41 =0.2 times 41 !!!! Which literally might be the pinnacle of school-math absurdity and really is a fantastically counterintuitive way to place the decimal.

The geometry section does not mention the infamous transversal and parallel line litmus test (WEHM p...)

Finally, there is this incredible jewel, in the category grades 9-12 that would seem to be precisely my book but isn't:

## The mathematicians Lament

amazon ranking: #150 mathematics study and teaching

It is nice to see that there is at least one other person who sees the school-math *content* as problematic. I'm kidding, everyone (except a handful of "experts") I've ever talked to basically agrees with me, they just never thought about it much, or how bad it really is big picture.

So this author's whole point is how absolutely terrible, Nay!, harmful school-math is.

"no pain will be spared to make the simple seems complicated"-p.85

This immediately jumped out at me and I could not agree more! The problem is the whole book is just that-a rant. And what a rant it is indeed! Going places like "masturbatory quadratic factoring" that I would just love to second but unfortunately the purpose of the WEHM manual is to elucidate, to shine light into the crevices, to light a candle in the dark so to speak rather than curse the darkness.

What is somehow exemplary of the whole situation is the fact that this is a teacher with many years on the job. Not only that, but apparently this is a teacher who was previously a real professor and a genius as well! So this is the one person, asides from me (that I currently know of), that escapes the catch 22 of either not knowing school-math or not knowing real math. But bizarrely he does not seem to care about actually helping the students, instead he writes a 300-page rant that has no practical advice. He seems to be advocating for a whole different species of mathematics and has no interest in helping anyone through the current system. Somehow I find it typical that the students are, once again, the last thing to cross anybody's mind.

There is no mention of PEMDAS or the vertical line test or parallel lines and the transversal etc, quadratic factoring is criticized but there is no advice or the student.

## <u>Online</u>

## Mathantics, YouTube (1.27 million subscribers)

Following the recommendation of the layman (or perhaps it was Mr. Mathantics himself?) that wrote that fascinating takedown of "the big fat math notebook" as a comment (p...) I checked out the YouTube channel *Mathantics*. Judging by this thoughtful and meaningful comment I expected to find some real quality here.

The presentation is quirky and funny with plenty of amusing roleplaying to make this something children definitely enjoy more than most drab math classes. But, true to the overall pattern developing here, the actual mathematical content is *catastrophic*. It is amazing how often attempts to do math differently end up making matters much, much worse.

Where to start I thought, this will take a while. So, I started at the beginning with adding and subtracting fractions. Since this is a topic that I have not included in my sample of the *WEHM manual* yet, allow me in one paragraph to summarize the problem school-math causes here.

When adding/subtracting fractions the denominators must be the *same* size as you cannot count pieces of pizza unless they are the *same* size. School-math hardly emphasizes this actual *explanation* of *why* denominators must be the same. Instead students are fed silly memorization drills such as: the fractions must *like* each other, so they must be *like* fractions (as used in this video). On the other hand, when multiplying fractions, you do *not* need to make the denominators the same. Many, many students can never *quite* remember which one, addition or multiplication, needs the common denominator. I am forever explaining the *why* of this this to 11<sup>th</sup> grade SAT students. A huge number of them never really understood this at all, they just memorized it and are consequently constantly forgetting it.

Going by the commentator's words criticizing the "big fat math notebook" and extolling Math Antics, this video should take care of such a common confusion.

For me, this is the hardest thing. Okay, you flip the fraction. But why? How does it work? Because this goes unsatisfied, I feel like I don't get it, and out of my mind it goes. Lots of folks are this way. I need to see how it works, and then I get that "ah ha!" feeling and it just sticks.

In this video, in order to be amusing or *seem* different (p...) I suppose, the following statements are made:

- 1. We at Math Antics are going to go through multiplication before addition of fractions.
- 2. The reason we cannot add fractions with unlike denominators is the Order of *Operations*.
- 3. Luckily, there is a *trick* to get around this. When the denominators are the same, we just add the top and keep the bottom.

### Where to begin?

The reason we cannot add fractions with different denominators is that you can't count pieces of pizza unless they are the same size. If the *Order of Operations* was in any way a clear explanation

of this<sup>9</sup> how do you explain your *trick*? It is absolutely, emphatically not a *trick*! It is *common sense* if explained correctly using pizza.

Judging by what happened in the video I am inclined to believe the words in the comment were not those of a layman, but that it was actually advertising by the channel.

Remarkable. I am always surprised by the ability to passionately pay lip service to some good cause and then, inexplicably, be incapable of even understanding the meaning of one's own words, especially in education where children are at stake.

I did not look at any other videos by this channel.

Now some could argue that the mere fact that a channel with such mediocre or even misleading content is succeeding at some level is proof of the fact that "nobody cares" (market analysis, p...). In other words, *mathematical* content is not what's in demand. Perhaps content is irrelevant and it's all about the entertainment value, to get kids through math somehow.

But this is not entertainment. Maybe one year everybody will buy a brand-new type of pink skis that glow in the dark. But unless they are also are good skis, they will be out of business by next season. Now, I realize this channel may continue forever, may even keep growing. What I'm saying is, it's doing this *in spite* of its mediocre content. If all the other types of skis were equally as bad as the bad pink skis, the pink skis would probably do quite well.

Score for this channel: new, original type of crazy. Extremely harmful do not leave in the reach of children.

<sup>&</sup>lt;sup>9</sup> Of course the Order of Operations is also true here, but that doesn't explain anything except that another rule doesn't contradict this rule. To make any real sense of it this way, to explain the "trick", you would even have to use the distributive law applied to fractions! Hardly intuitive.

#### Kahn Academy 5.3 million subscribers

Certainly not crazy. Kahn Academy is probably the closest thing out there to my book. It is almost the exact opposite of the very gimmicky Math Antics (see above.) Mathematically sound and serious, Khan academy really does take the time to explain what they are doing and does not just dump results on students to be memorized. They even shy away from a few of the worst school-math abuses, but by no means even close to every time. Interestingly, when they do "shy" away from these abuses in terminology, for instance, Khan Academy simply ignores the fact that in school the students are still confronted with these ridiculous terms and must answer questions using them (see below). This approach may help some students understand these ideas better, but does little to attack the real confusion being hoisted on these poor children in school, the confusions they must deal with every day, on every test. Many students never get past the terminology traps and other obstacles. In order to profit from this highly nuanced approach (when it does happen in a video), students must already be interested in actually understanding math, and so many probably don't find much comfort in this. It's as if the teachers on Khan academy are aware of the abuses in schoolmath (see below), but have been given a directive, or are personally disinclined, to call them out directly. This is no doubt because they do not wish to sound confrontational, to invite the ire of a whole swath of the "professional' class of educators, who will often defend this nonsense to their graves. Instead, even in these rare cases of "disrespect" for school-math, Khan Academy opts to gingerly step around the confusion rather than call it out.

This is really quite odd if you think about it. Why does a site that is dedicated to helping students worldwide, ostensibly not responsible to any DOE or standards, insist on *not* taking the side of the students? Why do they refuse to go there? Even though the students themselves have to deal with the non-sense and contradictory terminology and many other unnecessary confusions every day.

Why must the students fight with one hand tied behind their back? Why doesn't Khan academy really take the side of the students in these cases?

Again, this is only in very few, very select and obvious cases (such as the incredibly stupid parallel lines and transversal example, see below). In the vast majority of confusions, it seems, Khan academy still blindly repeats the school-math mantra, even going to ridiculous and extraordinary lengths to justify school-math nonsense (vertex of a quadratic, see below.) Or the videos are happily completely oblivious to the confusion at hand (dependent/independent variable).

It seems Khan Academy is neither fish nor fowl. A few relatively reasonable videos, a lot of terrible ones. Teachers with no courage to actually take the side of the student in this war on common sense.

I was also surprised at the low quality of some of the videos. Often the narrator is obviously riffing with no planning, and gets him-(all males so far)-self all confused, using the same variable twice in one picture for different things and getting quite involved and muddled, almost like a real classroom teacher. In this respect they could really take a page from the polished presentation of Math Antics (Ah, yes those glowing pink skis)

Here are a couple of quick examples to support my point. For more detailed treatments of the topics please read the page numbers given in the WEHM manual.

Khan Academy, for instance, fails to explain the *distributive property*, *dependent/independent* variable, *vertex of a quadratics*, what a *function* is and countless other topics in any meaningful way. And of course, there is my litmus test, the incredible *parallel lines and the transversal*:

### Parallel lines and the transversal, please read WEHM p... first!

This is actually only one of two examples I found where Khan Academy doesn't just blindly follow school-math's lead into the abyss. There were two different videos about it.

#### www.khanacademy.org-parallel-lines-and-transversals

What happens in this case, is they simply *ignore* the 5 impossible and contradictory names for angle pairs students must memorize in school and go straight to the way this *should* obviously be done.

At one point the narrator murmurs quietly that *vertical angles* should not really be called that because, well... they could just as well be horizontal (indeed!), but that is all the criticizing he seems to be able to muster of this absurdity. Asides from *corresponding angles* that are equal (because that's what *corresponding* means, he says or implies repeatedly), and a quick mention of *alternate interior angles*, the other angle pair names are not even mentioned. If you have read the WEHM entry on this, you know that the problem is that *corresponding interior angles* are *not* equal and *corresponding* suddenly does *not* mean equal anymore, rather, it now means on the *same* side of the line (the transversal that is). Notice how by avoiding the term *corresponding interior angles* and instead choosing *alternate interior angles* (which does kind of make sense) the narrator gingerly sidesteps the contradiction contained in school-math's lunatic approach of 5 names for 2 angles.

Imagine you are a student confused precisely by this terminology. If you were to stumble on this explanation which seems to smooth over the whole mess *as if* there were no problem-you would probably either start doubting your own faculties, or roll over, play dead and just give up trying to make sense of mathematics forevermore.

As mentioned above, the video also quickly becomes quite entangled. The narrator names both a point on a line *and* an *unrelated* angle **A**. This happens for a number of variables which he then starts putting equal signs in-between, as if that explains *anything*. He seems to notice his error, but plows ahead anyway, losing focus all the while. At another juncture, around 4:30, the narrator gets the feathers in his bonnet all twisted (making the very strange statement that there is *no* proof for this<sup>10</sup>) trying to explain that two parallel lines make the same angle with a common third line. The way I put it in the WEHM manual: parallel *means* "going in the same direction" so two lines going in the same direction make the same angle with a third line.

#### www.khanacademy.org transversals-and-parallel-lines

This a better video. More numbers, less variables and unnecessary confusion. Here as well, the narrator seems to use *corresponding* to just mean equal, with no regards for the other school-math terms *corresponding exterior* etc., see above. There is *no* mention of all the names at all, except *corresponding* and *vertical*. One of school-math's earliest sins is repeated in the use of the phrase: angles are *congruent*<sup>11</sup>.

Of course, if students are not concerned about school-math, but rather the standardized test, or actually understanding math this is great. But in terms of being a school-math aid, helping a bewildered student (not an avid one), this really does leave the students in the dark about what they are going through.

<sup>&</sup>lt;sup>10</sup> So then it would be an axiom, which it isn't !

<sup>&</sup>lt;sup>11</sup> Angles are not shapes, they do not even have fixed lengths, how can they be congruent? It's in every textbook, so certainly not this teachers fault, but he is repeating it, even though he then finally does use an equal sign.

### dependent/independent variable, WEHM p...

There are two basically identical videos. Please see the WHEM manual where this confusion is explained. In a nutshell, the fact that *independent* and *dependent* mean nothing else than x and y respectively, and the fact that, since this is math not chemistry, we could just switch these two terms around at will, is not mentioned. Does the narrator even realize this? There is absolutely no sign that he does. He simply executes what school-math says with zero explanation or awareness of the potential confusion. There are confounding, *nonsensical* (in the true meaning of the word) exercises in 8<sup>th</sup> grade math that ask: miles, gas which is the *independent* which is *dependent* variable; making it seem as if teachers and textbooks really don't understand this fundamental fact *themselves*. The comment sections here are a riot of confused students debating exactly this non-topic in a vacuum in utter darkness-a terrible sight to behold! Khan Academy apparently does not react to its own comment sections-they remain happily oblivious to the confusion they are continuing to promote. This is death by a thousand cuts, one more thistle in the side of the already dazed and confused math student (can all this really just mean x and y, then why for heaven's sake the big deal?), slowly sapping their energy and remaining will to attempt to understand anything in math at all. Bringing even the most determined one inch closer to just giving up and memorizing.

www.khanacademydependent-and-independent

www.khanacademy.orgdependent-and-independent-variables

### vertical line test, WEHM p...

Almost as obviously ridiculous and simultaneously universally embraced as *parallel lines and the transversal, the vertical line test* is the second instance I could find where Khan Academy differentiates itself from everyone else I've reviewed by not going full throttle into this calamity. The *vertical line test* is simply *ignored* and only mentioned, almost under the narrator's breath (even though this is what students world-wide are being taught everyday), at the very end: "sometimes we make a vertical line...". There is no recognition whatsoever that this is really silly and eminently unnecessary.

The narrator stops short of calling the *vertical line test* out, but also does *not* offer a better way of understanding what's going on. Using such terms as: relation, every x corresponds directly to one y, and a bunch of dots and no line; this video at some point becomes so contorted and in itself confusing, that I can easily understand a student sighing, and turning right back to the *vertical line test* for a (false) sense of security. Once again, a completely simple and logical explanation is readily available if you actually understand what a function is, if you understand that a function is indeed a completely natural occurrence (does the narrator believe this?).

You simply cannot have two different amounts of money in one bank account at the same time, see WEHM p...

khanacademy.graphical-relations-and-functions

vertex, WEHM p...

Oh boy, this video really seals the deal in terms of giving Khan Academy the benefit of a doubt.

The terrible and unnecessary *completing the square* is presented as the best way to *understand* where/what a vertex is. This is an incredible claim that even I have in 20 years never heard<sup>12</sup> a teacher or textbook make<sup>13</sup>. Usually, school-math just makes students do it this way unnecessarily with *no* justification. On top of this the actual method to do this *easily* and (it turns out, see WHEM p...) to *understand* what's going on (-b/2a) is *derided* as being...I don't know...confusing?...or... "just" a formula.

Never mind not being totally honest about how ridiculous the *5 angle names* or the *vertical line test* is, here Khan Academy is going *out of its way* to justify one of school-math's most utterly unnecessary tortures, see WHEM, p...

https://www.khanacadefeatures-of-quadratic-functions

#### distributive property, See WEHM p...

Every math TED talk begins with a phrase like "we must make mathematics more tied to real life". The distributive/associative properties are perhaps the *sole* instance where this is not only really, really *true*, but even existentially *necessary* to grasping what mathematics is actually all about. Given all this, it remains forever mystifying how school-math absolutely *refuses* to use a real-life example to explain these two rules and to make absolutely sure that all students understand that math is just the rules of the marketplace written down in symbols.

Not to mention that it is incredibly embarrassing (for the entire system) to go to school for 12 years and not even know the answer to a question that any vendor in the world needs to know by the age of 8 in order to survive, even if school is not much more than an imaginary concept to them.

Numbers, symbols, dots, jelly beans...crickets.

khanacademy.distributive-property

<sup>&</sup>lt;sup>12</sup> The mere fact that even after 20 years I am continuously surprised by the new levels of intricacies and confusion swirling around school-math, should tell you about the vastness of this ocean that students are actually fighting for their mathematical life in, the sheer size of the fields in disarray to be plowed over and planted with actual explanations for the hungry students, for excellence, for profit!

<sup>&</sup>lt;sup>13</sup> Also, if you do insist on going all the way to the vertex from to explain the vertex then at least see it as a horizontal and vertical shift!

#### Other You Tube channels/online resources

From here I googled *best math channels you tube* and *best math-help channels you tube* which (interestingly) brought up basically the same lists of mostly math "interest" videos for people already interested in math, "the origins of calculus" etc. The big one in this category is *numberphile* with 3.5 million subscribers, and they do not do math-help at all. Indeed, asides from the two big ones *Khan Academy* and *Math Antics*, there was really not much help with school-math at all. I did find a video by *Mario's math help* (65K) that listed his 10 favorite math help channels that were all in the 60-200 K subscribers range except *JMT tutoring* with over 1 million and *don't memorize* with almost I million, see below.

To give you an idea of the dearth of reasonable online resources for math-help, when I googled "best math-help You Tube" the top result was a list generated by *educators.com* that had *Khan Academy*, *Numberphile* and this as number 2:

#### Mathademics 9.5K subscribers

Either by pressing the link above or by going to the site and looking for the video "shortcut to solving proportions" you will find an incredibly terrible video. The first option presented is the usual trick of cross-multiplying (a highly questionable tactic in-itself, WEHM, p..., but, oh boy, are we far away from even having *that* discussion right now) but this genius teacher has a trick within the trick! All this is of course taking place with absolutely no explanation of what this all has to do with actual algebra and doing the same thing to both sides of an equation. On top of that, this trick within a trick (in this particularly ill-chosen example) happens to get both 12 by cross-multiplying and have a 12 on the bottom of the other fraction. Since the whole gist of her "revelation" is mumbled under her breath, I at first thought she was claiming you would always get the same number here, which is of course ridiculous. In fact, what is being suggested is to divide the one 12 by the other 12 getting 1. It took me several attempts to even understand this useless piece of misinformation. If you are really going to get into this in such detail, then why not just learn to multiply both sides by the denominator- for crying out loud, really! At the end of the video the teacher basically despondently says:" Yeah...that's all I got", and yet this site is on the first list that came up on Google.

Another list that came up a lot under similar searches (not including the words "channel" or "You Tube") by *commonsense.org* has Desmos (a graphing program) and 10 other useful resources for school-math but only *Khan Academy* in terms of a math-help site that is not for advanced study by students already interested in math, i.e. nerds.

I soon realized that it was easier to search for a topic on You Tube, than to search for a channel and then search that channel for a topic. Also, users are more likely to do it his way as well.

I entered my litmus test *parallel lines and the transversal*. It is my experience that if a teacher/book/site can't phrase this in the most obvious manner (because it's *so* darn obvious, any kid can understand that the school-math approach is nonsense) then there really is no reason to believe they will do any better with more complicated topics such as quadratics, PEMDAS or calculus for that matter.

Incredibly (I am still in disbelief as I write this), every single video that came up under this search followed or even embellished on school-math's crazy 5 names for 2 angles labyrinth except, to their credit, Khan Academy (see above). We are talking world-wide here. There are videos from India to China to England about this.

Let's discuss the ones with the highest rankings and subscriber numbers. I actively searched for videos that do *not* use the school-math approach but, again, found nothing except Khan Academy.

don't memorize 936k subscribers consecutive angles instead of corresponding opposite vertical no alternate exterior etc crazy explanation of alternate indian? Normal explanation at the end

https://youtu.be/6RMN5Pf1fHU 360,000 views

marios math tutoring 65.9 K subscribers

consecutive angles instead of corresponding (online thing?), problem with consecutive no normal explanation congruent, congruent

https://youtu.be/3Ex7SpsA9MI 5 k views

JMT tutoring

Barnes and Nobles

# Higher quality school-textbooks

There are a limited number of math textbooks that are mathematically better than all of this so far. These are not math-help books and the main problem is that they are too hard, too dense and brief to be useful to the vast majority of students. They also do however suffer from the basic structural problems of school-math which they accept without comment. They either ignore completely how school-math does things and go ahead and do it very compactly the way a mathematician would (which leaves students in classrooms around the world high and dry when they are confronted with the non-sense of daily math instruction) or they restate school-math (mathematically correctly mind you) with no indication that what they are faithfully executing is absurd.

## Envision Algebra 1

On p. 63 there is a pretty good<sup>14</sup> example of why implementing these higher quality textbooks alone never moved the needle in math scores. Point -slope form is given its own chapter as if it were nearly as important conceptually (especially for 8<sup>th</sup> graders) as slope-intercept form.

## A quick review:

*Slope-intercept form*: Written as what  $8^{th}$  graders have so far been told a function looks like, a machine that produces Ys. Form clearly shows the slope **m** and the y-intercept **b**.

*Point-slope form*: algebraic manipulation thereof that no longer starts off with y=, thereby obscuring the important parameters slope and y-intercept and of course no longer looking anything like a function. The purpose of this confusing looking formula is to get an equation from a point and a slope that must then be rearranged into a slope-intercept form. This "avoids" having to find the **b** using information (point is on the line) from the slop-intercept form.

But it is fundamentally important to be able to start "from scratch" from the slope-intercept form! This is the first case of the unspeakably important concept of using information (point is on the line) to find unknowns (the y-intercept). This concept becomes more and more central to mathematics the further you progress, starting next year in 9<sup>th</sup> grade with finding a quadratic from given information, (there is, thankfully, no point-quadratic form!) It is in fact possibly the very essence of mathematics, and also a mathematical concept that can be explicitly viewed as actual wisdom (What is it you want to know, #unknowns? What is it you know, #pieces of information?). I cannot even guess at the number of students I have had that definitely did not understand how to do this later (all the way to college) precisely because the point-slope form avoided it for them at the beginning.

<sup>&</sup>lt;sup>14</sup> I can see many education professionals defending point-slop to the death even some mathematicians may fondly recall it and in some rare advanced settings it may actually save a (negligible) step or two. Probably they will claim it strengthens algebra, which it most definitely does not do either. The admittedly quite solid algebra skills that are needed to get this monstrous creation back to slope-intercept form are quite lengthy for many 8<sup>th</sup> graders but also exactly the same every single time, so the poor students just memorize these steps as well. The sole advantage is not having to *think* about what a function is: "Oh, I have this point which I can plug into this function and then find an unknown"

In short, if you really understand *slope-intercept form* and how to find **b** it certainly won't kill you to use the point-slope formula over and over instead, but for everyone else-it will.

So instead of explaining what I just did to keep the teacher from repeating the mistakes he or she was taught, and helping students understand the purpose of math, this book simply states both forms in separate chapters as if they were of equal significance. Not only does this completely distorts the order of importance of concepts but what it really means in the hard reality of school-math is that teachers will simply skip to the *point-slope form* that they themselves were forced to memorize and barely touch on finding **b** from the original function. I can tell you that this has been the case with at least 85% of my hundreds of students.

What this represents in my estimation is one good example of how these better materials may not have outright mistakes and won't focus on illogical unnecessary stuff as much as the vast majority of school-math materials (see my examples) but they will certainly not call them out either. In fact, I could quite reasonably guess that including both of these forms on equal footing might be some form of political compromise. I wonder what the process is in terms of getting a schoolbook approved that actually goes against the grain of the usual course of school-math instead of just silently offering better ways but keeping the old roads open to traffic? The Common Core certainly does not say use point-slope form in 8<sup>th</sup> grade when functions are being introduced! So, where did it come from? Why is it still there?

Standard form:

## Envision Algebra 2

Extremely dense and not of much use to students, even gifted ones, as little to no explaining is done. Everything is more or less just stated quickly in its proper school-math order. There are few summary overviews or tables. Looking at my favorite benchmarks I see a lot of the same misplaced emphasis as in the rest of school-math. The main difference being that if you are already advanced in math you will find no technical faults with the content.

## Grouping with $a \neq 1$ (p.88)

In reality this is the single biggest consumer of most school-math class time in 9<sup>th</sup> grade. Here it is given exactly half a page with one barely worked (easy a=2) example and one incredibly compact inscrutable one-sentence paragraph. The confusing process of finding the factors of ac is not fully written out and also not commented upon. The whole process comes completely out of left field without any justification and makes no sense just as always in school-math. No mistakes, but also no reference to what a waste of time this really is and no acknowledgement that an explanation is missing here and that why this extended algorithm works every time is completely mysterious.

## Prime quadratics

Shifting functions

## Appendix I

### About completing the square

In many of these books that aim to explain school-math in a more down to earth fashion (for dummies, Danica McKellar, the Idiots Guide) an inordinate amount of space is taken up by *completing the square*, going into it in even more detail than school-math does, explaining it more carefully, sometimes even using geometric arguments. They treat this topic with great respect and afford it much importance. This might sound reasonable to someone completely alienated from the

everyday reality of school-math where *completing the square* is simply a pattern to be repeated ad nauseam with different numbers, and overwhelmingly blindly memorized-an academic catastrophe. Sure, this is how we get the mysterious *quadratic formula* and it is a neat trick that shows the importance of algebra, and yes it does come up *once* more in *one* very specific problem involving conic sections (circles and much less often ellipses) but we have to be realistic.

It is an odd idea for a school-math *help* book to delve even deeper into *completing the square* than school-math, making all kinds of arguments for the steps along the way as if explaining this academic catastrophe any further would help or interested the average student. After all, it is the average student that buys math-help books.

The reality is students have to either understand *completing the square* using the *binomial theorem* or, unfortunately far more often, *blindly* memorize the whole thing, including the secret term. There is no room in a math-help book for further discussion of this topic, it is useless for most students. To see why, read on.

Completing the square is long (7 steps) and tricky to *memorize*. In the ideal case we have a highly motivated student, not yet broken down by 10 years of school-math nonsense. This student hopefully has a full grasp on the *binomial theorems* and is interested enough in math to even care where the *Quadratic Formula* comes from. In this student's case, *completing the square* is relatively easy to follow as a result of said *binomial theorem*. This kind of student would pretty much immediately see how it works and what purpose it serves. This student would like to see this process repeated at most 3 times, perhaps applied to variables to get the *quadratic formula*. Granted, there might be a few of *these* students who would enjoy approaching *completing the square* in an alternative fashion as these books do. But that is precisely my point, these are not your average students. Of course, the last thing any mathematically endowed/interested person would want to do is what school-math actually does: keep needlessly repeating the long, process for months. Since we now have the final result of that whole process, the *quadratic formula*, this is really unnecessary.

On the other hand, the disinterested perhaps floundering student, just hanging on for dear life will probably not even understand the *algebra* of *completing the square*. Neither school-math nor these help-books use the *binomial theorems*<sup>15</sup> to explain what is going on here, instead it is taught as an *isolated* trick to be *memorized* including a secret term<sup>16</sup>. Consequently, your average student gains nothing here but memorized confusion.<sup>17</sup>

<sup>16</sup> They actually memorize  $\left(\frac{b}{2}\right)^2$  or  $\left(\frac{b}{2a}\right)^2$  as the secret term to add instead of mentioning the binomial theorem from which you can easily reason each time.

<sup>&</sup>lt;sup>15</sup> Once again WEHM,p...

<sup>&</sup>lt;sup>17</sup> Against the backdrop of an ocean of unexplained and misrepresented concepts (see the rest of my book) it actually seems mighty ridiculous to suddenly latch onto this particularly gruesome explanation and insist everyone suddenly become obsessed with understanding and proving everything, granted it is the quadratic formula. To put it more bluntly, from the left-to-right rule and PEMDAS or was it (PEDMAS?) to a picture with 5 names for 2 angles (*parallel lines and the transversal*) to endless meaningless factoring (it is not revealed until much later that this is how you *sometimes* get the x-intercepts) students have been trained to not ask too many questions. Now suddenly in 9<sup>th</sup> grade, the horrible *completing the square* is taken as an opportunity to reverse all that and become excruciatingly precise? Right. In school-math this becomes a downright torture of blind repetition and the help books start explaining it on an even deeper level than school even though this is no service to the struggling or average students.

Finally, the idea that many of these not so mathematically inclined students would hang on to this memorized pattern until 11<sup>th</sup> grade and conic sections, until that *one* exercise with circles finally rolls around again (pun intended), shows just how far from school-math reality these help-texts are. And, again, those students who actually are understanding this in any meaningful way, will understand it without much explanation if they use the *binomial theorem* to see where the middle term comes from (WEHM,p...)-which none of these books do, they memorize the secret term. (check)

## Odds and Ends

Math-Dictionary-Homework-Help-Families

This is the same book except grades 3-7.

Math-Refresher-Adults-Mastering-Essential

#58 mathematics

This is again only about very low level math, basically arithmetic. Also the author goes full force into the "math is all around you" mantra (WEHM, p...) that really isn't true and is, in my opinion, the beginning of the end of an honest relationship between math teacher and student. check

## Math-Inspectors-Story-Claymore-Diamond

A cute idea, once again only for elementary and beginning middle school. Also judging from the reviews it's really more of a story leading to the usual exercises, than math explanation.

## Can-Handle-Mindful-Mantra

This book is actually a psychological aid and does not give math advice at all. It is remarkable that this comes up under "math help" on amazon. This makes it abundantly clear just how much of a psychological burden math can be. Goes towards my thesis of math trauma.

## Math-Concepts-Everyone-Should-Learn

Way down in the amazon ranking and a general math enthusiasm book not geared to help students.

Everything-Parents-Guide-Common-Grades by everything by everything

#144 mathematics references 6-8

Every-Day-Math-Practice-Questions

Help\_Your\_Kids\_With\_Maths no link

Math-Youll-Ever-Need-Self-Teaching

#163 in mathematics review

Homework-Grown-ups-Everything-Learnt-Promptly

#143 literature encyclopedia

impress your friends or handle home work without humiliation. Not only math.

Help-Your-Kids-Math-New

#53 in mathematics study and teaching #13 STEM

## For adults

Help Your Kids with Math is the perfect guide for every frustrated parent and desperate child, who wants to understand math and put it into practice.

brain quest