IX. Independent and Dependent Variables

In mathematics (unlike in an actual physical experiment) all **variables** can be viewed as either **independent** or **dependent**. These two terms are, for mathematical purposes, *interchangeable*. Instead of explaining this, school-math suddenly introduces the terms **independent**/**dependent variable** in *8/9th* grade (long after functions have been already introduced), as if they were somehow centrally relevant to mathematics, which they *absolutely* are not.

In a chemistry experiment you may want to change the **pressure** in a tube to see how it affects the **temperature**. In this scenario it makes perfect sense to call the temperature the **dependent variable**, as it physically *depends* on the pressure you are applying. You are actually changing the pressure to see what happens to the temperature. In the case of a *physical* experiment the terms **independent**/**dependent variable** make a lot of sense.

From a mathematical point of view, however, it does not matter *who* is doing *what* in an experiment. If I change the pressure then the temperature changes. But we could reverse the experiment to make the pressure depend on what temperature is desired. In other words, in order to attain a certain temperature I would need to apply a certain pressure. In this scenario the pressure would be the **dependent variable**. A chemistry experiment could very well be set up that way around.
Mathematically speaking, independent/dependent variable just comes down to what you decide to put on the **x-axis** (**independent variable**) and what you put on the **y-axis** (**dependent variable**) when you make a graph! That's all.
In other words, as far as mathematics is concerned:

independent and **dependent variable** are simply other words for ***x*** and **y** respectively

This is hard for students to fully believe when they are confronted by this topic. Any rational 8/9*th* *grade* student assumes there must be a good reason school-math is now suddenly using these terms *instead* of the *x* and *y* that have been standard since functions were introduced at least a year ago around 7th grade. There must be some *hidden* reason these terms are important enough to now be introduced as a new topic. Well, aren't you silly for thinking that, there is no reason!

There is no mathematical reason why the temperature or pressure should be more dependent or independent than the other. In other words, as far as math is concerned, it could be either way around. It makes *absolutely* no difference. This whole topic is unnecessary in a math class, and it certainly is confusing (so mission accomplished, I guess.) During the initial *introduction* of **functions** in *6th*/7th grade it would be good to explain what a function is using these terms and the physical experiments as *examples* (they don't, see *p...*), but this should *end* in x and y being introduced as the dependent/independent variable. Once we have established how a function works (that you put the x in to get out the y) it is completely backward, and a sign of profound misunderstanding of the abstract nature of mathematics, to *backtrack* to these terms associated with physical experiments.

So, just when students begin to recover from the attack on their intellect due to the presentation of **functions** (*What is a function?, The vertical line test*, *p...*), just when they finally regain a sense of confidence and enough willpower to continue soldiering through; school-math comes along with this very unnecessary and damaging detour. This sends kids on a topsy-turvy *name-game* that, while obviously relevant in chemistry and physics, completely misses the point in mathematics.

Not only do these terms unnecessarily confuse students when introduced this way, but, as you will see below, school-math actually twists this into a very serious violation of the very basis of mathematical understanding.

Just when I have finished reassuring my students that the terms **dependent** and **independent variable**,

have no other meaning beyond *x* and *y,* it turns out that school-math itself does not understand that one **variable** *cannot*, mathematically, be more intrinsically **dependent** or **independent** than the other:

Pairs of **variables** are presented out of the blue and (unbelievably) students are asked which one is the **dependent** and which one is the **independent variable**.

As *if* there were (there *is not*) some mathematical reason why

"*pressure* is more **dependent** on *temperature*"

than

"*temperature* is **dependent** on *pressure*"

For example:

Determine the **dependent** and the **independent variable**:

Exercise 1

pressure, temperature

Exercise 2

money, popcorn

Exercise 3

gas, miles

Now school-math has gone beyond a mere confusing, unnecessary *word game*. This is an actual violation of one of the most fundamental truths in mathematics:

If *y* is **related** to *x*, then *x* is **related** to *y*

The real ear scratcher here is the practical issue of what to write down in school in order to get the *points* for this absurd question type.

In order to get the "right" answer, you will need to remember the exact example the teacher gave in his/her class. Was the number of miles dependent on the gas ? Or was the gas dependent on the number of miles in this *particular* example?

Often it's just random, i.e however the teacher happens to feel is more "logical" that day.

This is a waste of time and money, or worse.

These are real examples, to be found everywhere in school-math. For 20 years I've been amazed at what I come across in *homework assignments*, *textbooks* and s*tudents' notes*.

I don't blame you if you can't believe this yet. Please, check out this gem from the highly regarded *Khan Academy*[1]*:*

The video is about $y=5x$ and what is "more" independent, the *y* or the *x*! The narrator apparently does not understand (or for some reason prefers to ignore) that you could *divide* the equation by *5* on both sides and then it would just be the other way around:

$$x=\frac{y}{5}$$

I just watched this again with a student and had a quite a time scoffing at the endless comments section of poor extremely confused students.

If you had any doubt as to the point this book is making, I submit to you this comments section. It's so telling, so par for the course, it could be used as evidence in a congressional hearing about the chaos that school math breeds. This is why the entire *content* of the school-math curriculum needs to be *re-written.* Please note that I'm not saying there is anything wrong with the curriculum, the list of topics to be covered is just fine.

1 https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-dependent-independent/v/dependent-and-independent-variables-exercise-example-1

It is the *presentation* of the material regardless of what *order* it is placed in that is desperately in need of revision and quite frankly, *scandalous!*

This is just one more example of the nettlesome game of school-math. There is a window at the very beginning of **functions** in *6/7th* grade where it would be absolutely necessary to give concrete examples of what a **function** is and isn't (see: *Vertical Line Test, p...* and *What is a Function?, p...*) At this early stage the concept of a **dependent** and i**ndependent variables** are useful as part of the story of what functions are, how they are a very *natural* thing. But *6/7th* grade is not when these terms come up in school-math. In fact, for some reason, the opposite happens, and this name game mostly takes place far too late and in the wrong place: in *8th* grade, long after functions have already been introduced. This is odd because the *Common Core* has **dependent and independent variable** as a *6th* grade topic.

Of course the absurd exercises for this topic would be just as bad in *6th* grade (*p...*).

If you google these exercises they do come up everywhere, even in the most recognized school-math textbooks. Some are set up in a manner that makes it more or less clear which variable is supposed to be the dependent/independent one. But then there is a whole kaleidoscope of confusion- from completely free of meaning, to open to interpretation, to just slightly dubious, and so on. I see it all the time in my daily lessons.

Remember, all it takes is *one* bad problem and students are already confused, possibly *forever*. Math is hard enough. If school-math is patently incorrect on top of that, then it's basically *hopeless*.

The text-books and exercises don't even contain a hint of the notion that any such dependent/independent problem could just be *reversed*. They simply do not acknowledge this fact. Students tell me stories about how they are admonished by their teachers for daring to suggest a dependent/independent problem could just as well be the other way around. This is very, very sad and goes against the basic philosophical point of mathematics, that there is no subjectivity and that you should trust your own thoughts if they are logical!

As this example proves, just having a good curriculum is not sufficient. Math teachers need to be educated differently, so they don't just repeat bad practices.