VIII. Vertical Line Test

also see the chapter: *What is a Function?*

Having completely confused the students natural understanding of what a **function** is (*p...*) *6th* grade school-math now proceeds to further bury any hope of grasping what is actually going on in the topic of functions. This time the culprit is a superfluous concept named the **vertical line test**. This *procedure-with-a-name* once again replaces any real understanding of what is going on in mathematics. The **vertical line test** first comes up in *6th* grade and then obviously never really goes away just as **functions** never go away. If students don't understand this concept *without* memorization by *8th* grade functions (and math all together) will be very difficult to follow!

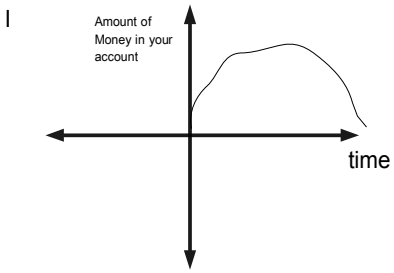
The question that the **vertical line test** is supposed to answer is:

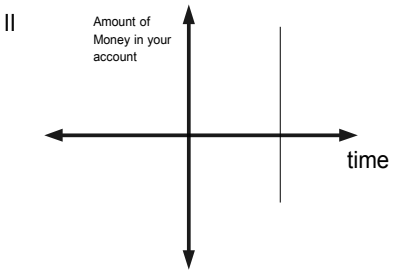
What kind of **graphs** can *not* be a **function**?

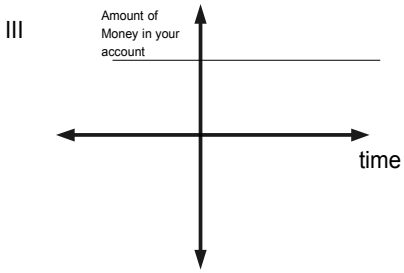
To best understand the rest of this entry, I suggest refreshing your memory by reviewing the entry *What is a Function?*, *p...*

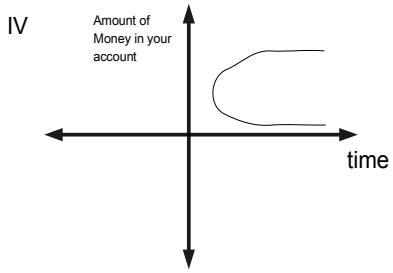
A *time vs. money* graph that shows how much money is in a bank account at a different times is one of the best examples of what a function is. Using this example of a function, the question that is supposed to be answered by the **vertical line test** becomes simply:

Which of the following graphs (*I-IV*) can and which can not represent the amount of money in a bank account at different times?









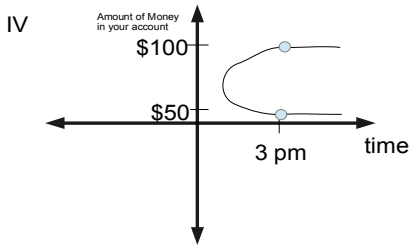
If the student has understood the concept of a function, this is the best moment to pause and let them play with the idea a little.

Please, don't just tell them the answer right away!

It's actually amazing how many students say something like: "*Graph IV* is not a function because time is going backwards" (too many sci-fi movies.) Then I say: "OK. Yes, but what exactly is the problem with *graph IV*? Can you explain a little more clearly why this can't happen?" After a little coaxing they often exclaim:

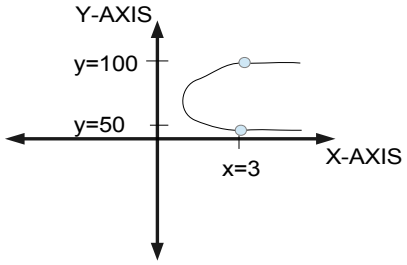
"Ahh!.....you can't have two *different* amounts of money in your account at the *same* time! I get it!"

seen on the graph:



You can't have *both $50* and *$100* in your account at *3 pm*!

The simple natural beauty of this "discovery" makes it absolutely clear why a function cannot have more than one **y-value** for any given **x-value**. *Because you cannot have more than one amount of money at any given time in a bank account, or* (thinking of the blood pressure monitor in every emergency room scene) *you can't have two different heartbeats at once!*



This explanation is *common sense*. Most importantly, it explains what is actually happening in mathematics without sounding abstract and intimidating.

Here's an actual quote from a typical school-math book:

"a function is a relation from a set of inputs to a set of *possible* outputs where each input is related to exactly one output. "

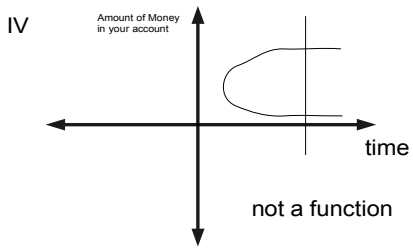
Other books use similar or even worse formalities. The use of the word *possible* here, for instance, is really just "cruel and unnecessary".

One would think this would be a great teachable moment because it solidifies the understanding of a function. But no. Enter, the **vertical line test**.

the vertical line test:

Instead of asking the student to think about what a **function** actually means (see above) students are instructed to make a **vertical line** through the **graph**.

If the vertical line cuts the graph in *more* than one point, then the graph is *not* a function.



This is, yet again, memorization replacing understanding, over-complicating matters and losing sight of what's really important. The perfect school-math trinity.

And no, **the vertical line test** is not a time saving trick students are told about *after* having fully understood an explanation like the bank account example above. School-math rarely even touches upon the the actual *concept* of a function when "explaining" the vertical line test. I know this from the hundreds of middle school students I have tutored in the last 20 + years. I know it sounds unbelievable, but if they *did* explain the basic idea, why would it even be necessary to make up a misleading *procedure-with-a-name*? It's totally obvious that you can't have two *different* amounts of money in an account at the *same time*! What further is there to say to an 8^{th} *grader* about that? **The vertical line test** also does not in any way save time, quite the opposite.

One of the more amusing pitfalls of the **vertical line test** is that later on in functions, there is actually something called the **horizontal line test**, which refers to **inverse functions** (*p...*). Since the emphasis is put on memorization and not a true understanding of what is going on, students most certainly could (and do) confuse the two! How are you supposed to remember if its *horizontal* or *vertical* if you don't understand the *why*?

I sometimes tell my older students that the **vertical line test** makes about as much sense as a *jump-off-the-building gravity test*. That sometimes gets a laugh.

The vertical line test is one of the worst offenses to *common sense* in school-math.

It is basically a direct attempt to *distract* from the *concept* of a **function**.

Because functions are everywhere in math, this often has catastrophic consequences

for the entire duration of a student's mathematical education.

And herein lies the trouble with school-math. The simplest, most logical explanation is hardly ever what your kids learn in school. The thought process among the creators of the curriculum must no doubt go something like this:

Why settle for a natural explanation when you can come up with a fancy *procedure-with-a-name* for something that should be completely obvious through common sense?

For the record, this is what the much villanized **Common Core** has to say about this:

"Understand that a function is a rule that assigns to each

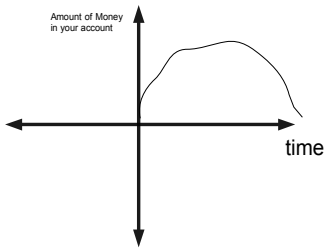
input exactly one output..."

The **Common Core** does not mention the **vertical line test**.

It's (mostly) *not* the **Common Core**, it's the **textbooks**.

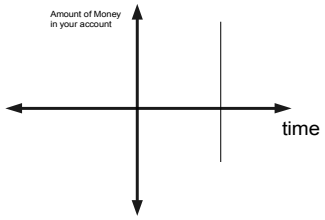
Notes on the graphs:

I



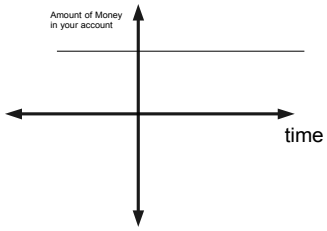
Graph I is a function, no problems here. You never have two amounts of money at the same time.

II



Graph II would mean you have *every* possible amount of money in your account at the *same time*, so not a function. This example can be confusing for the vertical line test. You would be making a vertical line through a vertical line!

III



Graph III means you have the *same* amount of money at *all* times, so you are just not spending or receiving any money. You never have two *different* amounts at the *same* time so therefore it is a function.