The easy way over

Find the vertex of $y = 2x^2 + 4x + 4$

The *x-coordinate of the vertex* is given by:

$$\frac{-b}{2a} = \frac{-4}{2(2)} = \frac{-4}{4} = -1$$

(The *general form of a quadratic* is always $y = ax^2 + bx + c$.)

Now you have the *x-coordinate*.
To get the *y-coordinate* you just:

plug the *x-value into* the *function*!!

$$y = 2(-1)^2 + 4(-1) + 4 = 2 - 4 + 4 = 2$$

And so,

$$vertex\ (x, y) = (-1, 2)$$

Comments

1.
The formula:

$$x\text{-coordinate of the vertex} = \frac{-b}{2a}$$

is usually presented as:

$$axis\ of\ symmetry = \frac{-b}{2a}$$

Many students never realize that these two things are identical. Often there are two questions on a test:

   a) find the *axis of symmetry*
   b) find *the vertex*

Students use $\frac{-b}{2a}$ to solve a) but then go through the 7 steps of *completing the*

The school-math way

Find the vertex of $y = 2x^2 + 4x + 4$

(The general form of a quadratic is:
$y = ax^2 + bx + c$)

Complete the square.
To do that first divide out the 2:

$$y = 2(x^2 + 2x + 2)$$

Now focus just on rewriting this part:
$$x^2 + 2x + 2,$$
So, a=1, b=2, c=2

It helps to add two blank spaces like this:

$$x^2 + 2x + \_ + \_ + 2$$

Add *and* subtract the "mystery" term:

$$\left(\frac{b}{2a}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$$

into the blanks above like this:

$$x^2 + 2x + 1 - 1 + 2$$

Now "magically" the first three terms are equal to a perfect square:
$$x^2 + 2x + 1 = (x + 1)^2$$
and so
$$x^2 + 2x + 1 - 1 + 2$$
becomes: $(x + 1)^2 - 1 + 2$

The whole process again:

$$
\begin{aligned}
y &= 2x^2 + 4x + 1 \\
&= 2(x^2 + 2x + 2) \\
&= 2(x^2 + 2x + 1 - 1 + 2) \\
&= 2[(x + 1)^2 - 1 + 2] \\
&= 2[(x + 1)^2 + 1]
\end{aligned}
$$

Or,

$$y = 2(x + 1)^2 + 2$$

The easy way over

square for b !?!

2.
The most amazing thing about this formula is that is already *inside the quadratic formula!!*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

See *vertex of a quadratic* section for an easy explanation why this is super obvious.

3.
If the instruction above:

> *"plug the x-value into the function!!"*

is not completely clear to you then you MUST read the section *what is a function?* even if your exam is tomorrow morning. Especially if your exam is tomorrow morning!

The school-math way

So, by *completing the square* we have finally arrived at the *vertex form of a quadratic*:

$$y = a(x - h)^2 + k$$

Where we know that the *vertex* is given by (h, k). We have:

$$y = 2(x + 1)^2 + 2$$

And so, the vertex is $(-1, 2)$

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comments
Completing the square is really important for math because that's where the *quadratic formula* comes from! The whole point of this amazing discovery in mathematics is that now have an amazing formula (see **easy way over**) and DO NOT need to keep repeating all these steps!! But try telling your teacher-am I right? You can probably use the **easy way over** method to get the answer and then just pretend to do *completing the square* and get away with it. Just don't tell them I said so.